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Construction of the High Order Accurate Generalized Finite Difference Schemes for Inviscid Compressible Flows

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Abstract. The construction of high order accurate generalized finite difference method for inviscid compressible flows still remains an open problem in the literature. In this paper, the high order accurate generalized finite difference schemes have been developed based on the high order reconstruction and the high order numerical flux evaluation on a local cloud of points. The WBAP limiter based on the secondary reconstruction is used to suppress oscillations near discontinuities. The implementation of high order accurate boundary conditions is of critical importance in the construction of high order schemes. A new method is proposed for the high order accurate boundary treatment. Several standard test cases are solved to validate the accuracy, efficiency and shock capturing capability of the proposed high order schemes.

AMS subject classifications: 35L25, 74J40, 76N15, 74S20

Key words: Cloud of points, high order accurate boundary treatment, high order schemes, generalized finite difference, shock capturing.

1 Introduction

The generation of a suitable mesh, especially for the complex geometrical configuration, is one of the major challenges in computational fluid dynamics (CFD). Geometries encountered in practical problems can be highly irregular and not strictly convex. Such cases are sometimes difficult to tackle using the conventional multi-block structured grid algorithms. A large amount of research has been devoted to develop the numerical methods based on unstructured grids or meshless approaches to overcome this problem. One

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such avenue of research is with the generalized finite difference (GFD) method, in which only clouds of points are used to construct the corresponding numerical schemes. GFD method represents a general class of numerical methods that can handle the problems with complicated geometries. Compared with the unstructured finite volume method, GFD method is easier to implement and there is no need to perform spatially numerical integration. When the clouds of points are extracted from the existing grids, GFD method may be considered as the finite difference method based on the unstructured grids. When the clouds of points are independent to any kinds of grid structures, GFD method is a certain kind of meshless methods. In the latter case, GFD method may fall under many other names including meshless [1–6], meshfree [7, 8], gridfree [9, 10], gridless [11–16], LSFD-U [17], and finite point [18–21] methods.

In the past two decades, extensive studies have been carried out in developing GFD or meshless methods for solving the compressible and incompressible flows. GFD method was firstly proposed by Chung [22]. Batina [11] developed an explicit gridless solver using a least-square curve fit on local cloud of points. Löhner et al. [19] used a linear polynomial to construct the flux distribution and adopted the method similar to one-dimensional MUSCL interpolation to compute the left and right states of the corresponding approximate Riemann solvers. Sridar and Balakrishnan [17] proposed an upwind least-squares-based finite difference framework for solving the inviscid compressible flows and used a linear fitting to approximate the spatial flux derivatives. Ding et al. [7] presented a third order scheme for the incompressible flows and investigated analytically the discretization error for derivatives. Shu et al. [23], Tota and Wang [8] used the radial basis functions to construct the flux derivatives. Katz and Jameson [1] compared three meshless schemes based on the Taylor series expansion, polynomial basis functions and radial basis functions. Su and Yamaoto [5] analyzed some behaviors of the meshless methods with a linear fitting. Sundar and Yeo [6] presented a high order meshless method with compact support for solving the incompressible flows. These methods are mainly first or second order in accuracy. While there have been some proposals for developing high order GFD method solving incompressible flows, very little work has been done on developing high order GFD method for compressible flows.

The purpose of this paper is to develop a high order GFD algorithm for solving the inviscid compressible flows. The construction of the present high order accurate GFD schemes consists of three basic steps. Firstly, a piecewise polynomial is recovered from the point values on the reference point and its support points (called stencil or cloud of points). Secondly, the numerical flux derivatives are computed in terms of the numerical fluxes on the mid-points (located halfway between the current point and its corresponding support points). Thirdly, the flow variables are updated with the time integration. The order of accuracy of the numerical flux derivatives depends on the reconstruction procedure and the number of terms included in the Taylor's series expansion for the flux evaluation.

The design of effective and robust limiters for the simulation of flows with discontinuities is a fundamental difficulty and is still an open problem for the high order accurate