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Local RBF Algorithms for Elliptic Boundary Value Problems in Annular Domains

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Abstract. A local radial basis function method (LRBF) is applied for the solution of boundary value problems in annular domains governed by the Poisson equation, the inhomogeneous biharmonic equation and the inhomogeneous Cauchy-Navier equations of elasticity. By appropriately choosing the collocation points we obtain linear systems in which the coefficient matrices possess block sparse circulant structures and which can be solved efficiently using matrix decomposition algorithms (MDAs) and fast Fourier transforms (FFTs). The MDAs used are appropriately modified to take into account the sparsity of the arrays involved in the discretization. The leave-one-out cross validation (LOOCV) algorithm is employed to obtain a suitable value for the shape parameter in the radial basis functions (RBFs) used. The selection of the nearest centres for each local influence domain is carried out using a modification of the kdtree algorithm. In several numerical experiments, it is demonstrated that the proposed algorithm is both accurate and capable of solving large scale problems.

AMS subject classifications: 65N35, 65N22

Key words: Radial basis functions, Kansa method, Poisson equation, biharmonic equation, Cauchy-Navier equations of elasticity, matrix decomposition algorithms, fast Fourier transforms.

1 Introduction

The local radial basis function (LRBF) method was first discussed in [30] and then independently introduced in [28, 31, 32, 34] introduced, see also [21]. In contrast to the traditional meshed based methods [1,2], LRBF is a meshless method which may be viewed as

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a special case of the Kansa method [15]. Meshless methods are well-suited for the numerical solution of boundary and initial value problems in two and three dimensions. Unlike the global Kansa-radial basis function (RBF) method [6–8] which leads to dense and poorly conditioned linear systems, the LRBF method leads to sparse systems. In recent years, the LRBF method has been successfully applied to a large variety of problems in science and engineering. No special treatment for pre-conditioning is required [22,23,33].

Efficient global Kansa-RBF algorithms for problems in geometries possessing radial symmetry were proposed in [18–20, 25]. These algorithms are *matrix decomposition algorithms (MDAs)* [4, 5] and make use of fast Fourier transforms (FFTs). This allows us to decompose a large system into a series of small systems which can be solved efficiently and thus the issue of ill-conditioning occurring for large dense matrices is resolved. However, when the number of collocation points is sufficiently large, the rank of the smaller decomposed matrices could still be large. In such a case, the memory space required to store these (not so small) matrices as well as the computational cost are still challenging issues. Furthermore, when the rank of the decomposed matrices becomes larger, the computational cost of finding a suitable shape parameter using LOOCV which is adopted in [25] increases rapidly. To alleviate these difficulties for very large-scale problems, a localized RBF method can be considered so that the decomposed matrices are sparse. Our goal in this work is to formulate the MDAs developed in [25] for the global Kansa-RBF method, for the LRBF method. As will be demonstrated, this leads to very efficient algorithms which exploit *both the structure and the sparsity* of the matrices involved, and thus to substantial savings in both computer time and memory. The nearest centres for each local influence domain in the LRBF method are selected using a modification of the *kdtree algorithm* [29]. While the emphasis of this paper is not on the determination of the optimal value of the shape parameter, we use the leave-one-out cross validation (LOOCV) algorithm [27] as a tool for determining appropriate values of the shape parameter which yield to satisfactorily accurate results. In the numerical examples examined in this paper, we shall focus on the use of the normalized multiquadric (MQ) while stressing that the proposed algorithms are applicable to other RBFs.

The paper is organized as follows. In Section 2 we present the three types of boundary value problems to be considered in the paper, namely Poisson, biharmonic and linear elasticity problems. Some important implementational issues related to the proposed technique are addressed in Section 3. A description of the LRBF method and corresponding MDA for Poisson problems is provided in Section 4 and its extension to biharmonic problems in Section 5. The LRBF method and corresponding MDA for linear elasticity problems is presented in Section 6. In Section 7 the proposed method is applied to several numerical examples and the results analyzed. Finally, some conclusions and ideas for future work are given in Section 8.

2 The problems

In all problems considered the domain Ω is the annulus