Commun. Comput. Phys. Vol. **24**, No. 2, pp. 557-575
doi: 10.4208/cicp.OA-2017-0082 August 2018

Mathematical Model of Freezing in a Porous Medium at Micro-Scale

Alexandr Žák[1,*], Michal Beneš[1], Tissa H. Illangasekare[2] and Andrew C. Trautz[2]

[1] *Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Trojanova 13, Praha, Czech Republic, 120 01.*
[2] *Center for Experimental Study of Subsurface Environmental Processes, Colorado School of Mines, Golden, Colorado 80401, USA.*

Received 4 April 2017; Accepted (in revised version) 3 November 2017

Abstract. We present a micro-scale model describing the dynamics of pore water phase transition and associated mechanical effects within water-saturated soil subjected to freezing conditions. Since mechanical manifestations in areas subjected to either seasonal soil freezing and thawing or climate change induced thawing of permanently frozen land may have severe impacts on infrastructures present, further research on this topic is timely and warranted.

For better understanding the process of soil freezing and thawing at the field-scale, consequent upscaling may help improve our understanding of the phenomenon at the macro-scale.

In an effort to investigate the effect of the pore water density change during the propagation of the phase transition front within cooled soil material, we have designed a 2D continuum micro-scale model which describes the solid phase in terms of a heat and momentum balance and the fluid phase in terms of a modified heat equation that accounts for the phase transition of the pore water and a momentum conservation equation for Newtonian fluid. This model provides the information on force acting on a single soil grain induced by the gradual phase transition of the surrounding medium within a nontrivial (i.e. curved) pore geometry. Solutions obtained by this model show expected thermal evolution but indicate a non-trivial structural behavior.

AMS subject classifications: 74N20, 74F10, 76S05, 80A22

Key words: Freezing, mechanics, phase-transition, soil, micro-scale.

*Corresponding author. *Email addresses:* alexandr.zak@fjfi.cvut.cz (A. Žák), michal.benes@fjfi.cvut.cz (M. Beneš), tillanga@mines.edu (T. H. Illangasekare), atrautz@mines.edu (A. C. Trautz)

1 Introduction

Improved understanding of the thermal, mechanical, and wetting behavior of soil or rock mass during freezing or melting is needed to address emerging environmental and industrial problems. These problems include: the design and maintenance of structures [1,2] in regions suffering from substantial temperature fluctuations; the exploitation of oil and gas resources in cold regions [3]; the underground storage of liquefied petroleum or natural gas within rock caverns [4], and the leakage of methane or carbon dioxide from melting permafrost into the atmosphere [5, 6].

A review of the literature related to these problems suggests that freezing-thawing behavior involves many complex processes (occurring during temperature shifts on the material surface) which consist of several effects that arise from the bulk nature of each material component, the interfacial interaction of the components, and the porous structure of the material. Individual effects can be identified as: (*i*) the temperature dependence of thermal and mechanical properties of constituents; (*ii*) the abrupt volume change during the phase transition of the wet medium; (*iii*) the surface tension between different phases; (*iv*) the drop of the freezing point of water associated with the surface tension effects; (*v*) the drainage of water pushed out of the freezing zones into air voids; (*vi*) the swelling of the soil material during freezing due to the suction of liquid water from unfrozen to frozen regions and the consequent growth of the ice mass; (*vii*) the movement of the ice-liquid interface during the cooling or warming of the material; (*viii*) the movement of the ice body; (*ix*) the opening of microcracks during freezing and the collapse of the void spaces during melting. Each of these effects contribute to the overall behavior differently under different conditions that are dependent primarily on the type of thermal setting (uniform or under a thermal gradient), its rate, soil material saturation, kind of soil/rock material, porosity, etc.

So far, several models have been developed that only partly describe the freezing-thawing behavior. They are usually created for specific scenarios under which some partial effects are allowed to prevail, and others neglected. This can be seen in the case of models that only consider the heat balance during the freezing and thawing processes [7, 8]. Other models deal solely with the frost heave [9–11], the upward movements of saturated soil with a high bearing capacity due to growth and movement of great masses of ice, under freezing conditions in the presence of a thermal gradient. The mechanism of frost heave has been identified by [12] and an explanation of the role of the premelted water in force action between soil constituents has been presented by [13]. In these models, cooling conditions are often assumed, and the swelling and movement processes are dominantly taken into account without regard for the volume differences caused by phase transition. However, in more general situations, volume changes of constituents can have a substantial influence on the bearing capacity [4].

Another shortcoming of the related models arises from their reliance on simplified assumptions or approximations of effects that are observed experimentally at the macroscopic scale. A lack of detailed microscopic considerations during model design may thus