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A Second-Order Path-Conservative Method for the Compressible Non-Conservative Two-Phase Flow

Yueling Jia[1], Song Jiang[1], Baolin Tian[1,*] and Eleuterio F. Toro[2]

[1] *Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing, 100094, China.*
[2] *Laboratory of Applied Mathematics, University of Trento, Trento, Italy.*

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Abstract. A theoretical solution of the Riemann problem to the two-phase flow model in non-conservative form of Saurel and Abgrall is presented under the assumption that all the nonlinear waves are shocks. The solution, called 4-shock Riemann solver, is then utilized to construct a path-conservative scheme for numerical solution of a general initial boundary value problem for the two-phase flow model in the non-conservative form.

Moreover, a high-order path-conservative scheme of Godunov type is given via the MUSCL reconstruction and the Runge-Kutta technique first in one dimension, based on the 4-shock Riemann solver, and then extended to the two-dimensional case by dimensional splitting. A number of numerical tests are carried out and numerical results demonstrate the accuracy and robustness of our scheme in the numerical solution of the five-equations model for two-phase flow.

AMS subject classifications: 35Q53, 35E15, 74C05, 74F10, 74M20

Key words: Two-phase flow, non-conservative form, hyperbolic equations, Riemann Solver, path-conservative approach.

1 Introduction

Multi-phase flow models and their numerical simulations have found a wide range of applications in science and engineering, such as environmental disciplines, the oil industry, nuclear engineering, self-propagating high-temperature synthesis, deflagration to detonation transition in combustion theory, etc. The underlying physics of the problems is complex and the aim of the mathematical models is to account for the behavior of at least two phases or fluids, and the interactions due to exchange of mass, momentum

*Corresponding author. *Email addresses:* `tian_baolin@iapcm.ac.cn` (B. Tian), `jia_yueling@iapcm.ac.cn` (Y. Jia), `jiang@mail.iapcm.ac.cn` (S. Jiang), `eleuterio.toro@unitn.it` (E. F. Toro)

and energy. A large class of models are based on the continuum theory and make use of average quantities [1–4] inside each control volume, allowing us to know the amount of each phase in the volume but not the position of the interphases. Models in current use, when neglecting dissipative effects, consist of non-linear systems of first-order partial differential equations along with closure conditions. There are two main emphases in the study of numerical methods for multi-phase flows. One is the hyperbolic character of the equations because hyperbolicity is an essential requirement for well-posedness [1] obtained. The other is the conservative character of the governing equations, i.e. a known conservation-law form in the mathematical sense, because one invokes physical conservation principles. that is to say, Mathematically, shock waves and the associated Rankine-Hugoniot conditions can be defined once the equations exists a conservative form. As we know, it is a quite challenging problem to develop the suitable and feasible exact or approximate Riemann solvers for multi-phase flow models due to the absence of hyperbolicity or conservation.

There are two kind of models for the compressible two-phase flows. One is the Saurel-Abrall model, another is the Baer-Nunziato model. The two models differ only in modeling the interface pressure and velocity. For the former model, the mixture velocity and pressure are computed by using the mass average of two phases, while for the latter the mixture pressure is taken as the pressure of the gas phase, and the velocity as the velocity of the solid or liquid phase.

Development of numerically accurate and computationally efficient algorithms for multi-phase flow simulations remains one of the unresolved issues in computational fluid dynamics. To our knowledge, almost all multi-phase flow models in the literature have non-conservative form due to the interface interaction. Examples include the Saurel-Abgrall model [5] and the Baer-Nunziato model [6], see also [1]. Given that most models in current use are in non-conservative form, it is of great importance to develop numerical methodology that can be applied to solve such systems of hyperbolic equations in non-conservative form. We also remark here that conservative hyperbolic models for multi-phase flows have recently been proposed [7], which are formulated in terms of parameters of state for the mixture.

In 1999, Saurel and Abgrall first presented their model for multi-phase flows and proposed a relevant Godunov-Rusanov method for interface problems between pure fluids and multi-phase mixtures [5]. The numerical scheme of Saurel and Abgrall is an extension of Rusanov scheme or HLL scheme from the conservative case to the nonconservative case, and no special Riemann solver is developed for their multi-phase flow model. Recently, Schwendeman and his co-workers developed a theoretical Riemann solver for the Baer-Nunziato model and a Godunov method based on the Riemann solver. They compared their scheme with other available methods in the literature, and the numerical tests show that the scheme in [8] is more accurate for the examples considered. Thereafter, Deledicque and Papalexandris established another similar exact Riemann solver based on description of six distinct centered waves [9]. They also compared the accuracy and robustness of three known methods for the integration of the non-conservative