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A Second-Order Finite-Difference Method for Compressible Fluids in Domains with Moving Boundaries

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Abstract. In this paper, we describe how to construct a finite-difference shock-capturing method for the numerical solution of the Euler equation of gas dynamics on arbitrary two-dimensional domain Ω, possibly with moving boundary. The boundaries of the domain are assumed to be changing due to the movement of solid objects/obstacles/walls. Although the motion of the boundary could be coupled with the fluid, all of the numerical tests are performed assuming that such a motion is prescribed and independent of the fluid flow. The method is based on discretizing the equation on a regular Cartesian grid in a rectangular domain $\Omega_R \supset \Omega$. We identify inner and ghost points. The inner points are the grid points located inside Ω, while the ghost points are the grid points that are outside Ω but have at least one neighbor inside Ω. The evolution equations for inner points data are obtained from the discretization of the governing equation, while the data at the ghost points are obtained by a suitable extrapolation of the primitive variables (density, velocities and pressure). Particular care is devoted to a proper description of the boundary conditions for both fixed and time dependent domains. Several numerical experiments are conducted to illustrate the validity of the method. We demonstrate that the second order of accuracy is numerically assessed on genuinely two-dimensional problems.

AMS subject classifications: 76M20, 65M06, 65M20, 76N99

Key words: Compressible fluids, Euler equations of gas dynamics, ghost-cell extrapolation, moving boundaries, finite-difference shock-capturing methods.

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1 Introduction

Development of accurate and efficient shock-capturing methods for hyperbolic systems of conservation laws has been a very active field of research in decades. Among the various methods, the ones that are most widely adopted are Discontinuous Galerkin (DG), finite-volume (FV) and conservative finite-difference (FD) schemes. DG and FV schemes are very flexible, since they can be naturally built on unstructured, possibly highly nonuniform meshes and thus can be applied on arbitrary domains. On the other hand, conservative FD schemes are easy to be made high-order in a very simple and efficient manner on Cartesian grid in several space dimensions. The main reason for their efficiency is that each flux derivative (converted into a flux difference along the coordinate directions) requires just one-dimensional (1-D) interpolations [30].

In view of the above considerations, our goal is to develop a simple and robust second-order conservative FD scheme for hyperbolic systems in the domains with fixed and moving boundaries. We concentrate on just one model—the Euler equations of gas dynamics. The methodology, however, could be also applied to other problems. In the two-dimensional (2-D) case, the governing equations are the compressible Euler equations:

$$\begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \end{pmatrix}_t + \begin{pmatrix} \rho u \\ \rho u^2+p \\ \rho uv \\ u(E+p) \end{pmatrix}_x + \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2+p \\ v(E+p) \end{pmatrix}_y = \mathbf{0}, \tag{1.1}$$

where ρ is the density, u and v are the velocities, E is the total energy, and p is the pressure. The system (1.1) is closed using the equation of states (EOS), which in the case of a polytropic gas reads as

$$E = \frac{p}{\gamma-1} + \frac{\rho}{2}(u^2+v^2). \tag{1.2}$$

We also denote by $c := \sqrt{\gamma p/\rho}$ the speed of sound, which will be used throughout the paper.

A simple FV method based on a Cartesian grid for the 1-D and 2-D compressible Euler equations in domains with solid moving boundaries was introduced in [8]. This FV method is an extension of the interface tracking method proposed in [7] for compressible multi-fluids. The key steps of the FV method from [8] are: (i) dividing all of the computational cells into the three groups: internal (fully occupied by the fluid), external (either located outside of the fluid domain or fully occupied by the solid obstacle), and boundary ones (partially filled by the fluid); (ii) evolving the solution in the interior cells only; (iii) replacing the unreliable data in the boundary cells with the data obtained by the solid wall extrapolation followed by the interpolation in the phase space. A slightly different approach was adopted in [18], in which arbitrary domains with moving boundaries were modeled using the level set method [25,27] and a FV method on a Cartesian mesh for compressible Euler equations in one, two and three space dimensions was proposed.