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Simulating Magnetohydrodynamic Instabilities with Conservative Perturbed MHD Model Using Discontinuous Galerkin Method

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Abstract. In magnetically confined plasma research, the understandings of small and large perturbations at equilibrium are both critical for plasma controlling and steady state operation. Numerical simulations using original MHD model can hardly give clear picture for small perturbations, while non-conservative perturbed MHD model may break conservation law, and give unphysical results when perturbations grow large after long-time computation. In this paper, we present a nonlinear conservative perturbed MHD model by splitting primary variables in original MHD equations into equilibrium part and perturbed part, and apply an approach in the framework of discontinuous Galerkin (DG) spatial discretization for numerical solutions. This enables high resolution of very small perturbations, and also gives satisfactory non-smooth solutions for large perturbations, which are both broadly concerned in magnetically confined plasma research. Numerical examples demonstrate satisfactory performance of the proposed model clearly. For small perturbations, the results have higher resolution comparing with the original MHD model; for large perturbations, the non-smooth solutions match well with existing references, confirming reliability of the model for instability investigations in magnetically confined plasma numerical research.

AMS subject classifications: 65M60, 65Z05, 85-08

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1 Introduction

Numerical simulation of magnetohydrodynamics (MHD) system is always an important subject in magnetically confined plasma research. One of the critical challenging issues of

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simulating laboratory plasma system is the macroscopic instability analysis in complex geometry, e.g. tokamak and stellarator. In principle, the target of the tokamak or stellarator, aiming for reactor, is to operate the plasma under control as a quasi-steady state system, i.e. equilibrium with perturbations. Large perturbations may lead to confinement degradation or even disruptions. On the other hand, small perturbations are universal and many of them can grow up to large perturbations in long-time operation. Thus the understandings of small and large perturbations are both critical for plasma controlling and maintaining steady state operation. The perturbations are usually of scale comparable with the experimental devices and such macroscopic dynamics of the magnetically confined plasma system can be described by MHD models. Up to now extensive investigations using MHD models have been carried out both analytically and numerically. However for very small perturbations in numerical simulations, the numerical errors generated from the equilibrium part may overwhelm the small perturbations if solving original MHD equations directly. Among fusion community, a widely used way is to split variables in original MHD equations into equilibrium part and perturbed part, resulting in non-conservative compact form MHD equations of perturbations, eliminating numerical errors from equilibrium. Existing simulation codes, such as NIMROD [1] and CLT [2], applying this non-conservative perturbed MHD model, together with classical finite difference methods and finite elements methods, can give good results when solving some instability problems in small perturbations stage, like KH instability and tearing modes. However when small perturbations grow up into strong nonlinear regime, this non-conservative model may lead to unphysical numerical results that break the conservation laws after long time computation, or even end up with numerical disruption, limiting its applications considerably. Although much progress in nonlinear MHD physics in toroidal geometry with these non-conservative MHD model has been made [1,3], strong nonlinear physics such as disruption [4] are still far from clear and much more efforts are required. So the development of numerical tools for simulating MHD instabilities with high accuracy, robustness and geometrical flexibility is still a hot point in magnetically confined plasma research.

So far many numerical methods and codes for solving conservative MHD models in different geometries have been developed. The most widely used numerical methodologies are based on either finite difference or finite volume spatial discretization. The advanced high order finite difference methods like ENO [5] and WENO [6] type schemes are extensively considered to be simple, effective and easy for code developing, especially on a Cartesian mesh or a mesh that can be mapped onto a Cartesian mesh smoothly. However, due to non-rectangular cross-section of tokamak or stellarator, it is difficult to be applied on such complex geometry. The finite volume method can in general handle the geometry by dividing such space into unstructured meshes, but it is rather complicated to build up high order accuracy schemes [7,8] on complex stencils. The recently developed numerical framework of discontinuous Galerkin (DG) method is well designed by many authors [9–14]. It is in principle able to achieve arbitrary high order of accuracy on both structured and unstructured mesh and more geometrical flexibility can be acquired.