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A Compact Scheme for Coupled Stochastic Nonlinear Schrödinger Equations

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Abstract. In this paper, we propose a compact scheme to numerically study the coupled stochastic nonlinear Schrödinger equations. We prove that the compact scheme preserves the discrete stochastic multi-symplectic conservation law, discrete charge conservation law and discrete energy evolution law almost surely. Numerical experiments confirm well the theoretical analysis results. Furthermore, we present a detailed numerical investigation of the optical phenomena based on the compact scheme. By numerical experiments for various amplitudes of noise, we find that the noise accelerates the oscillation of the soliton and leads to the decay of the solution amplitudes with respect to time. In particular, if the noise is relatively strong, the soliton will be totally destroyed. Meanwhile, we observe that the phase shift is sensibly modified by the noise. Moreover, the numerical results present inelastic interaction which is different from the deterministic case.

AMS subject classifications: 60H15, 60H35, 37K05

Key words: Coupled stochastic nonlinear Schrödinger equations, compact scheme, stochastic multi-symplectic conservation law, energy evolution law, charge conservation law, soliton evolution, soliton interaction.

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1 Introduction

The propagation of optical solitons in a nonlinear dispersive optical fiber is governed by the well-known coupled nonlinear Schrödinger (CNLS) equations of the form [5]

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$$\begin{aligned} &\mathrm{i}u_t+\alpha u_{xx}+\left(\sigma|u|^2+\beta|v|^2\right)u=0,\\ &\mathrm{i}v_t+\alpha v_{xx}+\left(\sigma|v|^2+\beta|u|^2\right)v=0, \end{aligned} \tag{1.1}$$

where u, v are complex amplitudes or envelops of two wave packets, α, σ and β are the group velocity dispersion, selfphase modulation parameter and cross-modulation coefficient, respectively; x and t represent the spatial and temporal coordinates. Due to their intrinsic stability, optical solitons have been proposed to use as information carrier for the long-distance fiber optic communications and the optical signal processing. A major limitation for the soliton applications is the multi-soliton evolution and interaction. To understand the nature and consequence of soliton evolution and interaction, extensive theoretical and experimental investigations have been carried out [4,8,10,12–14].

In practical circumstances, stochasticity is common in optical soliton communications. Generally speaking, the stochasticity can be classified into two basic types: the homogenous and nonhomogeneous [3]. For the homogenous case, stochasticity is presented in the input pulse with random initial values; for the nonhomogeneous case, due to the factor such as the random perturbation of the dispersion term, which may be originated by random variation of the optical fiber parameters, etc. Indeed, [17] studies the stability of solitons in a fiber with randomly oriented birefringence, and observes that the soliton does not split even at high values of the average birefringence. Meanwhile, in [16], a nonlinear optical fiber with random birefringence is modeled by a system of NLS equations with stochastic coupling, and the effects of randomly varying birefringence on the dynamics of propagating optical pulses are analyzed. Recently, [15] investigates the dynamics of the bound vector solitons for the coupled nonlinear Schrödinger equations with the random optical fiber parameters, and finds that soliton switching is closely influenced by the amplitudes of stochastic perturbations. This paper is devoted to the study of the influence of a noise term as the potential of CNLS equations on the propagation and interaction of solitons. It may cause drastically changes of the qualitative behavior of the solitons. The stochastic system of our interest is

$$\begin{aligned} &\mathrm{i}\mathrm{d}u+\alpha u_{xx}\mathrm{d}t+\left(\sigma|u|^2+\beta|v|^2\right)u\mathrm{d}t=\varepsilon u\circ \mathrm{d}W(t),\\ &\mathrm{i}\mathrm{d}v+\alpha v_{xx}\mathrm{d}t+\left(\sigma|v|^2+\beta|u|^2\right)v\mathrm{d}t=\varepsilon v\circ \mathrm{d}W(t), \end{aligned} \tag{1.2}$$

where ε is a nonnegative and real-valued constant which denotes the amplitude of the noise. The $\circ$ in the right-hand side of (1.2) means that the product is of Stratonovich type, and W denotes a real-valued space-time noise.

In this work, we use numerical simulations to study qualitatively the solutions of (1.2). This leads to choose a method which allows high resolution, compactness and economy. Furthermore, numerical methods preserving the structure characteristics of equations should be much better in preservation of physical properties and long time