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Eigenvalue Solver for Fluid and Kinetic Plasma Models in Arbitrary Magnetic Topology

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Abstract. ArbiTER (Arbitrary Topology Equation Reader) is a new code for solving linear eigenvalue problems arising from a broad range of physics and geometry models. The primary application area envisioned is boundary plasma physics in magnetic confinement devices; however ArbiTER should be applicable to other science and engineering fields as well. The code permits a variable numbers of dimensions, making possible application to both fluid and kinetic models. The use of specialized equation and topology parsers permits a high degree of flexibility in specifying the physics and geometry.

AMS subject classifications: 35Pxx, 76Wxx

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1 Introduction

Most modern computational efforts for simulating the edge region of fusion plasmas employ time advancement to capture the nonlinear and turbulent evolution of the particles and fields. Over the past several decades, many such studies of boundary turbulence have been advanced, employing varying degrees of approximation and sophistication. Some relevant examples are given in Refs. [1–18]. These simulation codes employ either fluid models, gyrofluid models, or full kinetic simulation using particle-in-cell (PIC) or Vlasov fluid approaches all of which are implemented in the time domain. The plasma simulation community needs such tools for theoretical analysis, numerical experimentation, experimental modeling, and even for the hardware component design.

On the other hand, there is a small but significant class of important edge plasma problems which are amenable to linear and/or quasilinear analysis of partial integro-differential equations. These include source-driven problems and eigenvalue problems

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such as those arising in the computation of the growth rates of linear plasma instabilities. In addition, linear calculations can be used for convergence tests and to study stability regions in parameter space.

Time-evolution codes can be used for treating such problems, however there are advantages in using a non time-evolution approach. Indeed, consider a general time-evolution equation

$$df(\vec{x})/dt = \vec{F}(f(\vec{x}),\vec{x},t). \tag{1.1}$$

In the case where the problem is linear, or can be linearized, the right-hand side can be represented as a matrix M, i.e.,

$$df(\vec{x})/dt = \hat{M}(\vec{x},t)f(\vec{x}). \tag{1.2}$$

Furthermore we are assuming M(x,t)=M(x), with no explicit time dependence. In this case, for calculation of the linear instabilities in this system, one can calculate the eigenvalues and eigenmodes of the matrix M by the methods of linear algebra,

$$\hat{M}f = -\omega^2 f. \tag{1.3}$$

This approach has advantages over solving the problem by time-evolution: it is potentially much more efficient in terms of CPU-hours needed to solve a problem of given resolution, and it is capable of finding subdominant modes, i.e. modes with less than the maximum growth rate. Such modes, even if stable, can lend useful insights into the underlying physics. [19, 20] Exploiting the full potential of this approach does require that a sparse eigensolver is employed; if full matrix techniques are used instead, the computational cost will be orders of magnitude higher. In addition, it is the experience of the authors that eigenmodes of interest can be found most easily and efficiently using spectral transform techniques, which are discussed in greater detail in Ref. [29]. Of particular interest is the Cayley transform, which both greatly decreases convergence times and allows eigenmodes from nearly any part of eigenvalue space to be selected.

Similarly, one can consider the problem of linear response:

$$d\vec{f}/dt = \hat{M}(x)\vec{f} + \vec{g}(x,t), \tag{1.4}$$

where $\vec{g}(x,t)$ is the forcing term.

Instead of solving by time-evolution, one can take advantage of the linearity and assume a single frequency time-dependence, $\vec{g}(x,t) = \vec{g}(x)\exp(-i\omega t)$. Assuming no secular terms and assuming the homogeneous part of the solution decays in time, one can solve for the asymptotic stationary solution by solving the linear system

$$(i\omega\hat{I} + \hat{M}(x))\vec{f} = -\vec{g}(x). \tag{1.5}$$

Finally, linear codes can also play a role in understanding some classes of nonlinear problems which are amenable to quasilinear analysis. The appropriate biquadratic forms can readily be calculated once the linear eigenfunctions are known.