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ADI-FDTD Method for Two-Dimensional Transient Electromagnetic Problems

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Abstract. An efficient and accurate numerical scheme is proposed for solving the transverse electric (TE) mode electromagnetic (EM) propagation problem in two-dimensional earth. The scheme is based on the alternating direction finite-difference time-domain (ADI-FDTD) method. Unlike the conventional upward continuation approach for the earth-air interface, an integral formulation for the interface boundary is developed and it is effectively incorporated to the ADI solver. Stability and convergence analysis together with an error estimate are presented. Numerical simulations are carried out to validate the proposed method, and the advantage of the present method over the popular Du-Fort-Frankel scheme is clearly demonstrated. Examples of the electromagnetic field propagation in the ground with anomaly further verify the effectiveness of the proposed scheme.

AMS subject classifications: 65N06, 65N12, 65N15, 65N22

Key words: ADI-FDTD, interface boundary, stability and convergence analysis.

1 Introduction

Interpretation of electromagnetic data in complex geological environments depends on the multidimensional forward and inverse modeling, and the topic is of great interest to geophysics community. The finite-difference time-domain (FDTD) method first introduced by Yee [44] and Taflove [38] is now generally regarded as one of the most commonly used tools in the EM exploration applications. Oristaglio and Hohmann [28] used the DuFort-Frankel scheme to simulate 2D transient response to the shut-off of a line

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source. Lepin [24] extended the FDTD scheme into 3D cases by using the Fourier transform along the strike direction, in which a 2D problem was solved for discrete wavenumbers. Such model is usually referred as a 2.5D problem, and it performs well for a general 3D structures [36]. Wang and Hohmann [42] extended the FDTD scheme to 3D applications, where the DuFort-Frankel scheme was employed with a staggered-grid. The divergence condition of the magnetic field was imposed and a displacement current term was introduced to ensure the numerical stability. Commer and Newman [4] developed a parallel version for 3D applications. By transforming the Maxwell equation to another form which was less frequency dependent, Maao achieved an efficient implementation of FDTD computation [25]. Other works basing on the finite difference including the hybrid finite-difference method and parallel computing were reported in [45] and [34].

In addition to the finite difference (FD) method, the finite volume (FV) and finite element (FE) methods have also been frequently used. The work on FV method covers both the frequency domain [5,14] and time domain [15]. With the advantage of dealing well with complex geometric domains as well as complicated geologic interfaces, the FE method is very popular in time domain [16, 17] and in frequency domain [18]. Goldman et al. [10] applied the FE method in the spatial formulation for the 2D problem and the backward Euler method in the time-domain. Everett and Edwards [7] developed the finite-element time-domain (FETD) method to simulate the marine electromagnetic propagation in 2.5D case. Um et al. [41] developed an iterative FETD to investigate the diffusion behavior in 3D earth, where an adaptive time step doubling method was considered to reduce the computing time. Besides the time domain approaches, many work has also been reported in the frequency domain. Without the consideration of time step, it is particularly suitable for applying FE to 2D [23], 2.5D [20] and 3D [30, 40] problems. Recent development on the FE method in EM includes the edge-based FE method [3,26], multifrontal method [6], adaptive FE method [12, 31], parallel computation [21, 30] and other inversion related problems [11,32].

However, it is well known that the computing cost associated with FE method is very expensive. It is not a trivial task to generate a proper grid system, the more complex the earth structure is, the more cost there will be needed. Besides, since the resultant matrix in the FE method is frequently ill-conditioned, the solutions may require the use of direct methods [40, 41]. It is worth to note that the computational cost for a direct solver is $\mathcal{O}(N^3)$, therefore a tremendous amount of storage requirement and computing time are demanded.

Compared with the FETD approach, one attractive advantage of the FDTD algorithm lies in its straightforward implementation. It is feasible to implement an efficient FDTD code with limited computing and storage resource. Further improvements are possible by considering implicit FDTD because of their favorable stability condition as well as computing efficiency, such as ADI-FDTD, Symplectic-FDTD, EC-S-FDTD, etc [2, 5, 8, 9, 27, 37]. With its unconditional stability, the ADI method first introduced by Peaceman Rachford [29] and Douglas [19] could take larger time step than the explicit schemes. Moreover, it is easy to extend an ADI algorithm from 2D problems to 3D problems.