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The Role of Inertia and Dissipation in the Dynamics of the Director for a Nematic Liquid Crystal Coupled with an Electric Field

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Abstract. We consider the dynamics of the director in a nematic liquid crystal when under the influence of an applied electric field. Using an energy variational approach we derive a dynamic model for the director including both dissipative and inertial forces.

A numerical scheme for the model is proposed by extending a scheme for a related variational wave equation. Numerical experiments are performed studying the realignment of the director field when applying a voltage difference over the liquid crystal cell. In particular, we study how the relative strength of dissipative versus inertial forces influence the time scales of the transition between the initial configuration and the electrostatic equilibrium state.

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1 Introduction

Liquid crystal refers to a state of matter that exhibits free flow similarly to a liquid, but with certain crystalline properties commonly associated with solids. In the nematic liquid crystal state, the long axis of the constituent molecules tend to align. This results in long-range orientational order with no long-range correlation of the centre-of-mass. In the classical continuum theory, the configuration of a nematic liquid crystal is described by a velocity field and a director field.

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The behaviour of a finite sample of a liquid crystal under the influence of an electric field is of particular importance. When applying an electric field there is a competition between the boundary energy and the elastic and electrostatic forces. In the *Fréedericksz* transition, the liquid crystal cell will realign when the applied field (electric or magnetic) is above a certain critical threshold. These kinds of switching-phenomena under applied fields are of great importance because of the application to Liquid Crystal Displays (LCDs).

In this paper we will focus on the director field and use numerical experiments to simulate its dynamics under the influence of an electric field. In particular, we aim to quantify the influence of the inertia term in comparison with the dissipation term on the dynamics of the director when the electric field is switched on. The present model is derived from the Oseen-Frank elastic energy, the electric energy, and a dissipation function by the least action principle and the principle of maximum dissipation. The resulting equation can be seen as a special case of the classical Ericksen-Leslie dynamic equations. More precisely, for a planar director field ψ and an electric potential U depending only on one space variable x and time, we will derive the equation

$$\sigma\psi_{tt}+\kappa\psi_t-c(\psi)(c(\psi)\psi_x)_x-\frac{1}{2}d'(\psi)U_x^2=0, \tag{1.1}$$

where

$$c(\psi)=\sqrt{\alpha\cos^2(\psi)+\beta\sin^2(\psi)}, \quad d(\psi)=\varepsilon_0(\varepsilon_\perp+\varepsilon_a\cos^2(\psi)),$$

and σ is an inertial constant, κ is a dissipation coefficient, α and β are the bend and splay elastic constants, ε_0 is the vacuum permittivity, and $\varepsilon_\perp$ and ε_a are dielectric constants.

The current model is closely related to the variational wave equation

$$\psi_{tt}-c(\psi)(c(\psi)\psi_x)_x=0. \tag{1.2}$$

This equation was first introduced by Saxton [24], and has since been subject to a considerable amount of research, see, e.g., [8,10–12]. In the current context, (1.2) can be seen as a special case of (1.1) when there is neither an electric field nor dissipation. Also, Chen and Zheng [2] investigated the equations without electric field that include both inertia and dissipation.

Disregarding inertial terms is almost ubiquitous in the modeling of nematic liquid crystals, and herein lies the main interest of the present paper. The behavior of nematics under an electric field has been studied extensively during the last decades [3–6,23,25]. However, to the authors' knowledge, analytical and numerical investigations have mostly been conducted for the static equations to find equilibrium solutions or for the parabolic equations where the inertia of the director is neglected. As discussed by Gang et al. [7] and van Doorn [26], in many cases there are good physical reasons why inertial terms are neglected. However, it was early noted by Leslie [19] that the rotational kinetic energy might play a role when the director is subjected to large accelerations. More