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On the Anisotropic Gaussian Velocity Closure for Inertial-Particle Laden Flows

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Abstract. The accurate simulation of disperse two-phase flows, where a discrete particulate condensed phase is transported by a carrier gas, is crucial for many applications; Eulerian approaches are well suited for high performance computations of such flows. However when the particles from the disperse phase have a significant inertia compared to the time scales of the flow, particle trajectory crossing (PTC) occurs i.e. the particle velocity distribution at a given location can become multi-valued. To properly account for such a phenomenon many Eulerian moment methods have been recently proposed in the literature. The resulting models hardly comply with a full set of desired criteria involving: 1- ability to reproduce the physics of PTC, at least for a given range of particle inertia, 2- well-posedness of the resulting set of PDEs on the chosen moments as well as guaranteed realizability, 3- capability of the model to be associated with a high order realizable numerical scheme for the accurate resolution of particle segregation in turbulent flows. The purpose of the present contribution is to introduce a multi-variate Anisotropic Gaussian closure for such particulate flows, in the spirit of the closure that has been suggested for out-of-equilibrium gas dynamics and which satisfies the three criteria. The novelty of the contribution is three-fold. First we derive the related moment system of conservation laws with source terms, and justify the use of such a model in the context of high Knudsen numbers, where collision operators play no role. We exhibit the main features and advantages in terms of mathematical structure and realizability. Then a second order accurate and realizable MUSCL/HLL scheme is proposed and validated. Finally the behavior of the method for the description of PTC is thoroughly investigated and its ability to account accurately for inertial particulate flow dynamics in typical configurations is assessed.

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1 Motivation and objective

Two-phase flows constituted of a gaseous phase carrying a disperse condensed phase play a key role in many industrial and scientific applications e.g. spray combustion in Diesel engines or aeronautical combustors, soot dynamics, fluidized beds. In all these applications the disperse phase is composed of particles/droplets of various sizes that can possibly coalesce or aggregate, break-up, evaporate and have their own inertia and size-conditioned dynamics.

To describe the disperse phase, many strategies can be envisioned. In the present work, we consider the dynamics of the particulate phase in a statistical sense using a kinetic approach and we describe it using a Number Density Function (NDF). The NDF measures an ensemble average (over a given set of initial conditions) number of particles at a specific location in the phase space. The phase space is determined by the number of internal coordinates that describe the particle state: position, velocity, size, temperature, *etc*.. These variables evolve due to physical phenomena: transport, drag force, evaporation, heating, *etc*.. which are accounted for through a Williams-Boltzmann Equation (WBE) [76], also called a Generalized Population Balance Equation (GPBE) in other scientific communities (chemical engineering, aerosol science).

There are several strategies to solve this kinetic equation: a direct resolution in the full phase space through deterministic methods is too expensive and beyond reach in most practical cases. A second choice is to approximate the NDF by a sample of discrete numerical parcels describing particles of various internal coordinates through a Lagrangian-Monte-Carlo approach [1, 25, 37, 56]. It is called Direct Simulation Monte-Carlo method (DSMC) in [8] and is generally considered to be the most accurate method for solving this type of WBE; it is specially suited for direct numerical simulations (DNS) on canonical configurations since it does not introduce any numerical diffusion. However, the number of parcels required to achieve a satisfactory statistical convergence comes to be high in 3D cases, especially when a high number of internal coordinates is required, and such an approach is no longer suitable for practical applications.

To overcome this limitation, Macroscopic Eulerian Moment Methods offer a promising alternative. Instead of solving the NDF itself, the WBE is integrated over selected dimensions of the phase space, including in particular velocity [19, 40]. Moment equations are obtained with a new phase space of reduced dimension, for which deterministic methods of discretization are affordable and efficient. The coordinate of the phase space which is the most essential to deal with is the velocity, because it will drive the spatial distribution of particles. Thus in the following we will focus on distributions which are monodisperse in all variables except velocity. Additional dimensions of the phase space