

# Direct Numerical Simulation on Mach Number and Wall Temperature Effects in the Turbulent Flows of Flat-Plate Boundary Layer

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**Abstract.** In this paper, direct numerical simulation (DNS) is presented for spatially evolving turbulent boundary layer over an isothermal flat-plate at  $Ma_\infty = 2.25, 5, 6, 8$ . When  $Ma_\infty = 8$ , two cases with the ratio of wall-to-reference temperature  $T_w/T_\infty = 1.9$  and 10.03 are considered respectively. The wall temperature approaches recovery temperatures for other cases. The characteristics of compressible turbulent boundary layer (CTBL) affected by freestream Mach number and wall temperature are investigated. It focuses on assessing compressibility effects and the validity of Morkovin's hypothesis through computing and analyzing the mean velocity profile, turbulent intensity, the strong Reynolds analogy (SRA) and possibility density function of dilatation term. The results show that, when the wall temperature approaches recovery temperature, the effects of Mach number on compressibility is insignificant. As a result, the compressibility effect is very weak and the Morkovin's hypothesis is still valid for Mach number even up to 8. However, when Mach number equal to 8, the wall temperature effect on the compressibility is sensitive. In this case, when  $T_w/T_\infty = 1.9$ , the Morkovin's hypothesis is not fully valid. The validity of classical SRA depends on wall temperature directly. A new modified SRA is proposed to eliminate such negative factor in near wall region. Finally the effects of Mach number and wall temperature on streaks are also studied.

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**Key words:** Hypersonic, directly numerical simulation, compressibility effects, turbulent boundary layer, strong Reynolds analogy.

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## 1 Introduction

Generally speaking, friction resistance and heat flux along the out-side metallic layer (skin) of high-speed aircraft increase sharply when the boundary layer changes from

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laminar to turbulence. But there are two difficulties for drag reducing and heat degrading. One is how to predict transition region accurately, while another is to fully understand the inertial mechanism of compressible turbulent boundary layer (CTBL). The vortex structure and the inertial dynamics mechanism, which are closely related to CTBL, play a key role in the aero-industry [1]. The transition prediction is a popular subject of studying of boundary layer stability. The mechanism of CTBL which is another active studying area, will also be discussed in the present paper.

Direct numerical simulation (DNS) that involves the rapid development of computing technology is currently an important method in the study of turbulent mechanisms. In comparison with the results of experiment and theory, as what has been assessed by Schlatter et al. [2], DNS has the distinct advantage that all of the case-specific parameters (the inflow fields, boundary conditions and disturbances) can be set accurately, and no random measurement error corrupts the data that are obtained. High-order scheme always plays an important role in DNS of compressible boundary layer turbulence, especially at high Reynolds or with high Mach number. In order to numerically simulate such complex flow, various high-order and high resolute schemes [3–7] have been developed in past decades. No doubt, WENO [8–10] and its derived schemes are of the most successful ones. Especially, some low dissipative WENO type schemes have been proposed, such as compact-WENO [11, 12], WENO-Z [13], WENO-SYMO [14], TWENO [15], which have been successfully used for multi-scales capture. As Pirozzoli [16] reviewed and suggested that the hybridization of a high-order compact scheme with the WENO scheme is good choice for the DNS and larger eddy simulation (LES) of turbulent compressible flows. Moreover, it is still an arduous work to enhance the robust or stability of such low dissipative and high resolute methods, especially for the case of flow with high Mach number or high Reynolds number.

Recently, most of studies on the DNS for compressible turbulence focus on compressibility effects, especially on checking the validity of Morkovin's hypothesis [17]. This hypothesis indicates that, at a moderate free-stream Mach number (about  $Ma_\infty \leq 5$ ), the dilatation is small, and any differences from incompressible turbulence can be considered by the mean variations in the fluid properties. Hitherto, this hypothesis is the basis for the analysis of compressible turbulence.

The DNS results of the spatial simulations for the CTBL with  $Ma_\infty \leq 2.25$  over the flat-plate are proposed by Rai et al. [18], Pirozzoli [19], Gatski [20], and Li [21], respectively. The results show that the essential dynamics of the CTBL greatly resemble the incompressible case. Pirozzoli et al. [22] further to proposed the meticulous structure of a spatially evolving supersonic boundary layer by DNS with  $Ma_\infty = 2$  up to  $Re_\tau \approx 1120$ . Such a result provides possibilities to start probing the effects of high Reynolds numbers.

Maeder et al. [23] investigated the effects of the Mach number and the wall temperature by using temporal simulations for the CTBL with  $Ma_\infty = 3, 4.5, 6$ , corresponding to a isothermal wall with  $T_w/T_\infty = 2.5, 4.4, 7$ , respectively, over a flat-plate boundary layer. In his computations, the wall temperature approximates recovery temperature. The results demonstrate that Morkovin's hypothesis and the strong Reynolds analogy (SRA)