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Simulation of Inviscid Compressible Flows Using PDE Transform

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Abstract. The solution of systems of hyperbolic conservation laws remains an interesting and challenging task due to the diversity of physical origins and complexity of the physical situations. The present work introduces the use of the partial differential equation (PDE) transform, paired with the Fourier pseudospectral method (FPM), as a new approach for hyperbolic conservation law problems. The PDE transform, based on the scheme of adaptive high order evolution PDEs, has recently been applied to decompose signals, images, surfaces and data to various target functional mode functions such as trend, edge, texture, feature, trait, noise, etc. Like wavelet transform, the PDE transform has controllable time-frequency localization and perfect reconstruction. A fast PDE transform implemented by the fast Fourier Transform (FFT) is introduced to avoid stability constraint of integrating high order PDEs. The parameters of the PDE transform are adaptively computed to optimize the weighted total variation during the time integration of conservation law equations. A variety of standard benchmark problems of hyperbolic conservation laws is employed to systematically validate the performance of the present PDE transform based FPM. The impact of two PDE transform parameters, i.e., the highest order and the propagation time, is carefully studied to deliver the best effect of suppressing Gibbs' oscillations. The PDE orders of 2-6 are used for hyperbolic conservation laws of low oscillatory solutions, while the PDE orders of 8-12 are often required for problems involving highly oscillatory solutions, such as shock-entropy wave interactions. The present results are compared with those in the literature. It is found that the present approach not only works well for problems that favor low order shock capturing schemes, but also exhibits superb behavior for problems that require the use of high order shock capturing methods.

AMS subject classifications: 35K41, 65N99, 76L05, 76N15

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1 Introduction

Hyperbolic systems of nonlinear conservation laws

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = 0 \tag{1.1}$$

with an initial condition

$$\mathbf{u}(x,0) = \mathbf{u}_0(x) \tag{1.2}$$

have attracted great attention in the past few decades in mathematical, scientific and engineering communities due to their practical applications in fluid mechanics, aerodynamics, and nano-bio systems, to mention only a few. The solution to this class of problems may not exist in the classical sense because of possible discontinuities in the initial condition, material interface, singularity formation, turbulence, blow-up, etc.

Both global and local methods have been developed for hyperbolic conservation laws. Many up-to-date local methods have been proposed for shock-capturing, turbulence and shock interaction, including weighted essentially non-oscillatory (WENO) scheme [21, 23, 37, 38], central schemes [3, 27, 31, 34], arbitrary-order non-oscillatory advection scheme [43], gas kinetic [30, 35, 58], anisotropic diffusion [36], conjugate filters [19] and image processing based algorithms [17, 52]. An important factor that contributes to the success of the above mentioned local schemes in the shock-capturing is their appropriate amount of intrinsic numerical dissipation, which is introduced either by explicit artificial viscosity, upwinding, relaxation, or by local average strategy in non-oscillatory central schemes [24]. Indeed, the characteristic decomposition based on Roe's mean matrix can also be considered as a local average of the Jacobian matrix. The relation between some approximate Riemann solvers and relaxation schemes was analyzed by LeVeque [29]. Local characteristic decomposition is not necessary in low-order methods because of intrinsic numerical dissipation, while it seems to be indispensable in high-order methods [38]. In general, local and low order methods perform well for problems whose Fourier responses of the solution focus predominantly in the low frequency region. For this class of problems, first order or second order Godunov type of schemes can be very efficient in balancing accuracy and efficiency. When local and low order methods are used for resolving shocks in flows with fine structural details or highly oscillatory patterns, their numerical dissipation is usually too large to offer informative results.

Spectral methods, or global methods, on the contrary, produce little numerical dissipation and dispersion in principle when applied to approximate spatial derivatives. It is well known that spectral methods are some of the most accurate and efficient approaches for solving partial differential equations (PDEs) arising from scientific and engineering