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A Parallel Computational Model for Three-Dimensional, Thermo-Mechanical Stokes Flow Simulations of Glaciers and Ice Sheets

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Abstract. This paper focuses on the development of an efficient, three-dimensional, thermo-mechanical, nonlinear-Stokes flow computational model for ice sheet simulation. The model is based on the parallel finite element model developed in [14] which features high-order accurate finite element discretizations on variable resolution grids. Here, we add an improved iterative solution method for treating the nonlinearity of the Stokes problem, a new high-order accurate finite element solver for the temperature equation, and a new conservative finite volume solver for handling mass conservation. The result is an accurate and efficient numerical model for thermo-mechanical glacier and ice-sheet simulations. We demonstrate the improved efficiency of the Stokes solver using the ISMIP-HOM Benchmark experiments and a realistic test case for the Greenland ice-sheet. We also apply our model to the EISMINT-II benchmark experiments and demonstrate stable thermo-mechanical ice sheet evolution on both structured and unstructured meshes. Notably, we find no evidence for the "cold spoke" instabilities observed for these same experiments when using finite difference, shallow-ice approximation models on structured grids.

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Key words: Stokes-flow modeling, ice-sheet modeling, finite element approximation, finite volume approximation, parallel implementation.

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1 Introduction

During the past five years, there has been a concerted effort towards the development of improved numerical and computational models for glaciers and ice sheets. This is due to renewed concerns about the potential for future sea-level rise from land-ice melting [1–3] and to deficiencies in existing land-ice models, as highlighted by the last assessment report of the Intergovernmental Panel on Climate Change [32, 33]. A primary deficiency of the land-ice models used in that report was the simplified treatment of the ice-sheet dynamics which governs the three-dimensional velocity field within the ice. For glaciers and ice sheets, dynamical behavior is most completely and accurately described by a nonlinear Stokes system; recent papers [13, 14, 36] have reported the numerical and computational treatment of such models.

In [14], we reported on a new, nonlinear Stokes computational model, which used high-order accurate finite element methods on unstructured, variable resolution meshes. That work concentrated solely on the efficient and accurate parallel-computational solution of the Stokes momentum balance equations. Here, we mainly focus on important improvements in numerical approximations and solvers to that basically same computational model in order to make it more efficient and useful for practical science applications, in particular the simulation of large-scale, thermo-mechanically coupled ice-sheet evolution.

The remainder of this paper is organized as follows. In Section 2, we present the governing equations for ice-sheet dynamics and evolution of ice temperature and thickness in the Stokes thermo-mechanical model for simulating ice-sheet flow. Their numerical approximations and consequent solution techniques are then presented in Section 3. In Section 4, we test our computational model using standard diagnostic and prognostic experiments and compare our results with those from some previous models. Concluding remarks follow in Section 5.

2 The Stokes thermo-mechanical model for ice-sheet flow

The three-dimensional, thermo-mechanical Stokes ice-sheet model consists of three coupled components: the diagnostic, nonlinear Stokes equations governing the flow dynamics (conservation of momentum), the prognostic equation describing the evolution of the ice temperature, and the prognostic equation determining changes in the ice-sheet geometry (conservation of mass). See, e.g., [11,13,14,16,35,36], for additional details concerning the model we consider here.

Let $[0,t_{\max}]$ denote the time interval of interest and Ω_t the three-dimensional, time-varying spatial domain occupied by the ice sheet. The dynamic behavior of the ice-sheet is modeled by the Stokes equations for an incompressible, power-law viscous fluid in a