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An Implicit Algorithm of Solving Nonlinear Filtering Problems

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Abstract. Nonlinear filter problems arise in many applications such as communications and signal processing. Commonly used numerical simulation methods include Kalman filter method, particle filter method, etc. In this paper a novel numerical algorithm is constructed based on samples of the current state obtained by solving the state equation implicitly. Numerical experiments demonstrate that our algorithm is more accurate than the Kalman filter and more stable than the particle filter.

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Key words: Kalman filter, particle filter, implicit filter, Monte Carlo method, stochastic differential equations.

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1 Introduction

The main purpose of numerical simulations of a filtering process is to obtain, recursively in time, a good estimate for the probability density function (pdf) of the state of a dynamical system based on noisy observations. The first major breakthrough in this classical problem of signal analysis is the landmark work of Kalman and Bucy (Kalman filter) [16] on linear filtering (see also [9, 19, 20, 22]), under the assumption of linearity of the system and Gaussianity of the noise, and the conditional distribution of the state, given the observations, is Gaussian. This conditional distribution gives the best estimate of the

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statistical description of the state of the system based on all the available observation information up to the current time. Largely because of the success of Kalman filters, linear and nonlinear filters have been applied in the various engineering and scientific areas, including communications such as positioning in wireless networks, signal processing such as tracking and navigation, economics and business, and many others.

In most of the practical application problems, however, linearity assumption is not valid because of the nonlinearity in the model specification process as well as the observation process. Two of the widely used methods for nonlinear filtering problems are extended Kalman filter (EKF) [2,8,13–15] and particle filter method (PFM) [3–5,11,21,25]. In EFK the estimation problem is linearized around the predicted state so that the standard Kalman filter can be applied. The central idea of the particle filter method is to represent the desired pdf of the system state with a set of random samples. As the number of samples becomes very large, PFM provides a representation of the pdf. Since the seminal work of Gordon, Salmond and Smith [11], there have been significant development in both practical applications and theoretical analysis on PFM. Other efforts of solving nonlinear filtering problems include the Gaussian sum filter [1, 17, 24], moment methods based on approximations of the first two moments of the density [18] and Zakai filter which represents the pdf as the solution of a parabolic type stochastic partial differential equation (Zakai equation) [6, 10, 12, 23, 26, 27].

While the aforementioned methods have been remarkably successful in attacking the nonlinear filtering problem, each of them has its drawbacks and limitations. For instance, when the state equation describing the signal process and the observation equation are highly non-linear, the extended Kalman filter can give particularly poor performance. PFM has a number of advantages over EFK, including its ability to represent arbitrary densities, adaptively focus on the most probable regions of state-space. However, it also has a number of disadvantages, including high computational complexity, degeneracy for long period simulation and its difficulty of determining optimal number of particles.

The goal of this paper is to construct a new algorithm for numerical simulations of nonlinear filtering problems. The general framework of our algorithm is adopted from the Bayesian filtering theory which constructs the pdf of the state based on all the available information. We still use the general framework of Kalman filters and particle filters which solve the problem by two stages: prediction stage and update stage. For each time recursive step, the prediction stage gives the estimation for the prior pdf of the future state based on the currently available observation information while update stage gives the posterior pdf from the updated observation information and the result obtained in the prediction stage. However, instead of attempting to search for a representation of the pdf as in PFM, we approximate the pdf as a function over a grid in state space. Specifically, at the prediction stage, we attempt to seek the predicted pdf of the future state variable through a Monte Carlo method by evaluating the conditional expectation of the future state with respect to the current stage. Since the sample points for the current state is computed by solving the state equation implicitly, we name our method as an "implicit filter method". The following two items summarize the novelty of our approach: