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Shape Recovery of Elastic Capsules from Shear Flow Induced Deformation

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Abstract. Red blood cells undergo substantial shape changes in vivo. Modeled as a viscoelastic capsule, their deformation and equilibrium behavior has been extensively studied. We consider how 2D capsules recover their shape, after having been deformed to 'equilibrium' behavior by shear flow. The fluid-structure interaction is modeled using the multiple-relaxation time lattice Boltzmann (LBM) and immersed boundary (IBM) methods. Characterizing the capsule's shape recovery with the Taylor deformation parameter, we find that a single exponential decay model suffices to describe the recovery of a circular capsule. However, for biconcave capsules whose equilibrium behaviors are tank-treading and tumbling, we posit a two-part recovery, modeled with a pair of exponential decay functions. We consider how these two recovery modes depend on the capsule's shear elasticity, membrane viscosity, and bending stiffness, along with the ratio of the viscosity of the fluid inside the capsule to the ambient fluid viscosity. We find that the initial recovery mode for a tank-treading biconcave capsule is dominated by shear elasticity and membrane viscosity. On the other hand, the latter recovery mode for both tumbling and tank-treading capsules, depends clearly on shear elasticity, bending stiffness, and the viscosity ratio.

AMS subject classifications: 74F10

Key words: Fluid-structure interaction, shape recovery, lattice Boltzmann method, immersed boundary method.

1 Introduction

The shape change of viscoelastic, fluid-filled capsules has received considerable attention by researchers in recent years. This attention has particularly centered on its application to red blood cells, which may be modeled in such a way. The passage of blood through

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small capillaries requires significant deformation by red blood cells, from the normal biconcave discoid to a bullet-like shape [30]. Upon reaching larger blood vessels, the red blood cells recover their normal shape. In blood diseases, such as sickle-cell anemia, the ability of red blood cells to deform and subsequently recover their shape is reduced. As a result, they may block capillaries and oxygen delivery may be adversely impacted [28]. To aid the development of treatments for such blood diseases, and to better understand the mechanical structure of red blood cells, it is important to study the manner in which their shape is deformed and recovered.

Extensive study has been made of the deformation of viscoelastic, fluid-filled capsules under shear flow. Blood flow *in vivo* is not archetypal shear flow, but this has become a standard venue for considering the shape change of red blood cells. Experimental and theoretical work [8, 21, 22] has been recently complemented by significant computational simulations [5, 30, 31]. Among studies focusing on a single capsule, the constitutive law governing the elastic character of the membrane has been investigated [2]. The role of the membrane's bending stiffness has been studied by boundary integral [18] and lattice Boltzmann methods [25]. The effect of different capsule and ambient viscosities on deformation has been discussed [20, 26], and considered in concert with membrane viscosity [29]. Recently, a strikingly novel volume-of-fluid method was introduced which has the advantage of doing without a separate structural grid [11].

Conversely, investigations into the shape recovery of capsules from deformation have been largely limited to experimental and theoretical avenues. These studies primarily aimed at measuring the time course of shape recovery and determining the dominant mechanisms by which it occurred. Micropipette aspiration has been used to induce a large deformation in part of the red blood cell membrane and allow that deformation to relax [7]. Complementing their experimental work with a theoretical analysis of micropipette aspiration, Evans and Hochmuth characterized the time course of this recovery with the exponential decay function $e^{-t/tc}$. Their work concluded that the time constant is given as $tc = \frac{2\eta_e}{E_s}$, where η_e is the capsule's membrane viscosity constant and E_s is the capsule shear elasticity modulus [7]. More recently, optical tweezing has been applied to study the relaxation of red blood cell membranes and showed behavior consistent with the exponential decay function characterization [4]. In their associated theoretical model of optical tweezing, similar to that for micropipette aspiration, Dao *et al.* also characterized tc as a ratio of η_e to E_s. Lastly, red blood cells have been placed in shear flow and deformed until reaching an equilibrium state. After abruptly stopping the shear flow, the time course of the shape recovery has been measured by the same exponential decay function [3]. Among other shear flow studies, Fischer showed that red blood cells possess a shape memory [8] and Sutera *et al.* suggested that the time constant tc may also depend on the ambient viscosity [27].

While relaxation from micropipette aspiration or optical tweezing may be a primarily solid mechanical process, as modeled in [4] and [7], the shape recovery from cell deformation in shear flow suggests that a similar characterization may be used to describe a more complex case of shape recovery [3]. Red blood cell deformation in shear flow having been