Commun. Comput. Phys.
doi: 10.4208/cicp.210513.091013a

Vol. **15**, No. 5, pp. 1320-1342
May 2014

A Stability Analysis of Hybrid Schemes to Cure Shock Instability

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Received 21 May 2013; Accepted (in revised version) 9 October 2013

Communicated by Kun Xu

Abstract. The carbuncle phenomenon has been regarded as a spurious solution produced by most of contact-preserving methods. The hybrid method of combining high resolution flux with more dissipative solver is an attractive attempt to cure this kind of non-physical phenomenon. In this paper, a matrix-based stability analysis for 2-D Euler equations is performed to explore the cause of instability of numerical schemes. By combining the *Roe* with *HLL* flux in different directions and different flux components, we give an interesting explanation to the linear numerical instability. Based on such analysis, some hybrid schemes are compared to illustrate different mechanisms in controlling shock instability. Numerical experiments are presented to verify our analysis results. The conclusion is that the scheme of restricting directly instability source is more stable than other hybrid schemes.

AMS subject classifications: 35L65, 65M08, 76M12, 76L05

Key words: Godunov methods, numerical shock instability, hybrid scheme.

1 Introduction

In the last several decades, Godunov-type [6] schemes based on Riemann solvers are among of the most successful methods in computational fluid dynamics (CFD), which exhibit strong robustness in most situations. However, there are some problems in extending Godunov methods to two-dimensional case, for example, the Roe solver [28]

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and HLLC solver [31] for the Euler equations may suffer from carbuncle and odd-even decoupling phenomena that are called numerical shock instability [25]. Recently, similar phenomena have been found in shallow water simulations [11].

Several attempts have been made to understand and cure the phenomenon [4,12,18,22,25,29,35]. Quirk [25] gives a one-dimensional analysis and points out the relation between pressure and density perturbations, that is, a scheme will be afflicted by the odd-even decoupling if the perturbation to the pressure field affects the density field. He uses a pressure gradient to detect a shock, and constructs a hybrid method by combining the HLL [9] and Roe [28] schemes to cure shock instability. Pandolfi and D'Ambrosio [22] extend the analysis of the linearized algorithms to several upwind schemes, and then use the information obtained to design remedies to shock instability by slight and local modification to the original schemes (i.e., by imposing an additional linear wave). Gressier and Moschetta adopt similar analysis method [7] to derive a criteria which predicts the odd-even decoupling, and show that the strict stability on Quirk's test is incompatible with the exact resolution of steady contact waves. Sanders et al. [29] notice that the dimension by dimension extension of one-dimensional upwind schemes to the multidimensional equations of gas dynamics often leads to poor results when computing stationary shocks. Through a different linear analysis from Quirk's, they show that this failure is an instability which is the result of inadequate crossflow dissipation implied by strictly upwind schemes. A multidimensional dissipation based on one-dimensional entropy correction is provided to eliminate the instability. Other multidimensional effect is considered in rotational Riemann solver [3,15,20,26]. Ren [26] analyzes dissipative term of linear shear wave, and shows that compared with the grid-aligned flux function, the rotated flux function has more dissipation, and thus it has better stable performance. Liou [18] identifies that the multidimensional shock instability comes from a pressure term in the mass flux. Thus the AUSM schemes whose mass fluxes do not depend on the pressure term are free from carbuncle phenomenon [18,30]. Furthermore, Park and Kwon [23] point out that the existence of the pressure term in mass fluxes is only a sufficient condition for the scheme being shock-stable. Xu and Li discuss the dissipative mechanism in the Godunov method in [35]. They give an explanation of numerical shock instability by using a formula for quasi-one-dimensional nozzle fluid. Due to the numerical dissipation diminishing in transverse direction of shock wave, the numerical shock instability will appear. Especially in the subsonic region, once there is perturbation inside the numerical shock layer, the instability will happen. In the supersonic region the flow structure is essentially stable with respect to any small perturbation. Recently Jiequan Li et al. [16] compare difference of the generalized Riemann solver and the gas-kinetic scheme for inviscid compressible flow simulations, they regard when the non-equilibrium physical reality in shock layer is replaced by an equilibrium one, the shock instability will be triggered since the equilibrium state used inside a shock layer cannot provide enough numerical dissipation. Dumbser et al. develop a stability analysis for two-dimensional steady shocks based on the matrix method [4]. The background fluid is a steady shock wave instead of a uniform constant state as that in Quirk's analysis. The perturbation is