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# A High-Order Accurate Gas-Kinetic Scheme for One- and Two-Dimensional Flow Simulation

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**Abstract.** This paper develops a high-order accurate gas-kinetic scheme in the framework of the finite volume method for the one- and two-dimensional flow simulations, which is an extension of the third-order accurate gas-kinetic scheme [*Q.B. Li, K. Xu, and S. Fu, J. Comput. Phys., 229(2010), 6715-6731*] and the second-order accurate gas-kinetic scheme [*K. Xu, J. Comput. Phys., 171(2001), 289-335*]. It is formed by two parts: quartic polynomial reconstruction of the macroscopic variables and fourth-order accurate flux evolution. The first part reconstructs a piecewise cell-center based quartic polynomial and a cell-vertex based quartic polynomial according to the "initial" cell average approximation of macroscopic variables to recover locally the non-equilibrium and equilibrium single particle velocity distribution functions around the cell interface. It is in view of the fact that all macroscopic variables become moments of a single particle velocity distribution function in the gas-kinetic theory. The generalized moment limiter is employed there to suppress the possible numerical oscillation. In the second part, the macroscopic flux at the cell interface is evolved in fourth-order accuracy by means of the simple particle transport mechanism in the microscopic level, i.e. free transport and the Bhatnagar-Gross-Krook (BGK) collisions. In other words, the fourth-order flux evolution is based on the solution (i.e. the particle velocity distribution function) of the BGK model for the Boltzmann equation. Several 1D and 2D test problems are numerically solved by using the proposed high-order accurate gas-kinetic scheme. By comparing with the exact solutions or the numerical solutions obtained the second-order or third-order accurate gas-kinetic scheme, the computations demonstrate that our scheme is effective and accurate for simulating invisid and viscous fluid flows, and the accuracy of the high-order `GKS` depends on the choice of the (numerical) collision time.

**AMS subject classifications**: 76M12, 76M25, 76N15

**Key words**: Gas-kinetic scheme, initial reconstruction, BGK model, Boltzmann equation, velocity distribution function.

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## 1 Introduction

Over the past three decades there have been many research activities in the design and application of high order accurate numerical methods in computational fluid dynamics. High order methods are especially desirable for simulating flows with complicated solution structures. Up to now, based on the idea of Godunov method [3], many people developed various schemes based on approximate Riemann solver and various high-order accurate Godunov-type schemes and applied to some practical problems successfully. Some examples are MUSCL (monotone upstream-centered schemes for conservation laws) scheme [28], the total variation diminishing scheme [6], the essential non-oscillatory method scheme [7], the weighted essential non-oscillatory method scheme [16], the discontinuous Galerkin method [2], the spectral difference (SD) method [17], and spectral volume method (SV) [30] and so on. Upwind and high-resolution methods are well-reviewed in [29] and the high-ordered WENO schemes are summed up in [19]. We also refer the readers to the books [2, 14, 27] and references therein.

The aim of this paper is to construct a high-order accurate gas-kinetic scheme for one- and two-dimensional flow simulations from the viewpoint of gas-kinetic theory. The development of the gas-kinetic schemes including the KFVS [18] and BGK-type scheme [34] for compressible flow simulations has attracted much attention in the past two decade years. The gas-kinetic BGK schemes is a class of finite volume schemes to evolve the macroscopic variables and are constructed based on the solution of the Bhatnagar-Gross-Krook (BGK) model of Boltzmann or collisionless Boltzmann equation. It turns out that the gas-kinetic schemes have provided robust and accurate numerical solutions for various unsteady compressible flows, see [22–24] and references therein. Some theoretical analysis has also been carried out for the gas-kinetic schemes, see e.g. [25, 26], etc. Recently, a third order gas-kinetic Navier-Stokes solver with one dimensional flux evaluation has been presented in [15].

The paper is organized as follows. Section 2 introduces the BGK model. Section 3 presents our high-order accurate gas-kinetic scheme including two quartic polynomial reconstructions of the macroscopic variables and fourth-order accurate flux evolution. The generalized moment limiter is also implemented detailedly. Several 1D and 2D test problems are solved in Section 4 to demonstrate the effectiveness and accuracy of our proposed scheme in simulating invisid and viscous fluid flows. Conclusion is given in Section 5.

## 2 The BGK model

In physics, specifically non-equilibrium statistical mechanics, the Boltzmann equation or Boltzmann transport equation describes the statistical behaviour of a fluid not in thermodynamic equilibrium, i.e. when there are temperature gradients in space causing heat to flow from hotter regions to colder ones, by the random (and biased) transport of parti-