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High-Order Interpolation Algorithms for Charge Conservation in Particle-in-Cell Simulations

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Abstract. High-order interpolation algorithms for charge conservation in Particle-in-Cell (PIC) simulations are presented. The methods are valid for the case that a particle trajectory is a zigzag line. The second-order and third-order algorithms which can be applied to any even-order and odd-order are discussed in this paper, respectively. Several test simulations are performed to demonstrate their validity in two-dimensional PIC code. Compared with the simulation results of one-order, high-order algorithms have advantages in computation precision and enlarging the grid scales which reduces the CPU time.

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Key words: High-order algorithms, charge conservation, PIC code, CPU time.

1 Introduction

Particle-in-cell (PIC) codes are widely used in plasma physics and astrophysics because it is simple and straightforward. It is well known that PIC method can be carried out by solving continuity equation instead of Poisson equation [1].

There are several techniques for satisfying the continuity equation [1–7], which are called "charge conservation methods". In references [4–6], the authors introduced a charge conservation method for simple shapes of quasi-particles. As described in references [2, 3], the particle trajectories were divided into straight line segments between

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the start and end points. The current density was assigned to each segment and then charge conservation could be achieved for each particle trajectories. In references [1], the author developed a method of density decomposition in Cartesian geometry, which was a new charge conservation method. The method was valid for arbitrary form-factor of particles.

Umeda developed methods for first-order [7] and second-order [8] spline interpolation. In Umeda's methods, a particle trajectory was assumed to be a zigzag line in one time step. The methods could be used without any "IF" statements, which enhanced the speed of computation without any substantial distortion of physics. The method for first-order is widely used in PIC codes because of its simple and straightforward. The method used in second-order spline interpolation is not as simple and straightforward as the method used in first-order. It is well known that higher-order algorithms can reduce the numerical noises and increase the sizes of grid scales [9]. In this paper, we develop two new methods of higher-order algorithm for the condition of a particle trajectory assumed to be a zigzag line in one time step and can also be used without any "IF" statements to enhance the speed of computation, which are simple and straightforward. The methods can be applied to any even-order and odd-order, respectively. And the validity is checked by comparing the results of the two methods with one-order method.

This paper is organized as follows: in Section 2, the zigzag scheme for second-order which can be expanded to any even-order is presented. In Section 3, the zigzag scheme for third-order which can be expanded to any odd-order is considered. In order to check the usability of our algorithms, we compare the simulation results of high-order with the case of one-order in Section 4. The conclusions are summarized in Section 5.

2 Zigzag scheme for second-order spline interpolation

Let us consider the continuity equation in finite differences [1] and reduce it to two dimensions, which can be written as

$$\begin{aligned}\frac{\rho^{t+\Delta t}(j,k)-\rho^{t}(j,k)}{dt}&+\frac{J_x^{t+\frac{\Delta t}{2}}(j+\frac{1}{2},k)+J_x^{t+\frac{\Delta t}{2}}(j-\frac{1}{2},k)}{dx}\\&+\frac{J_y^{t+\frac{\Delta t}{2}}(j,k+\frac{1}{2})+J_y^{t+\frac{\Delta t}{2}}(j,k-\frac{1}{2})}{dy}=0.\end{aligned}\tag{2.1}$$

Here dx and dy are the grid spaces and dt stands for one time step. The charge density ρ is made up of form-factors of particles

$$\rho(j,k)=\sum_i q_i S_{j,k}(x_i,y_i).\tag{2.2}$$

Here q_i, $S_{j,k}(x_i,y_i)$ are the charge and form-factor of the ith particle. When a particle move from a location of (x^t,y^t) to another, which can be written as $(x^{t+\Delta t},y^{t+\Delta t})$ and they are