Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2018-0250

Vol. **27**, No. 3, pp. 861-896
March 2020

An Approximate Riemann Solver for Fluid-Solid Interaction Problems with Mie-Grüneisen Equations of State

Li Chen[1], Ruo Li[2,*] and Chengbao Yao[1,3]

[1] *School of Mathematical Sciences, Peking University, Beijing, China.*
[2] *HEDPS & CAPT, LMAM & School of Mathematical Sciences, Peking University, Beijing, China.*
[3] *Northwest Institute of Nuclear Technology, Xi'an, China.*

Received 21 September 2018; Accepted (in revised version) 15 November 2018

Abstract. We propose an approximate solver for compressible fluid-elastoplastic solid Riemann problems. The fluid and hydrostatic components of the solid are described by a family of general Mie-Grüneisen equations of state, and the hypo-elastoplastic constitutive law we studied includes the perfect plasticity and linearly hardened plasticity. The approximate solver provides the interface stress and normal velocity by an iterative method. The well-posedness and convergence of our solver are verified with mild assumptions on the equations of state. The proposed solver is applied in computing the numerical flux at the phase interface for our compressible multi-medium flow simulation on Eulerian girds. Several numerical examples, including Riemann problems, underground explosion and high speed impact applications, are presented to validate the approximate solver.

AMS subject classifications: 76L05, 74H15, 76T30, 68U20, 47E05

Key words: Fluid-solid interaction, Riemann solver, hypo-elastoplastic, Mie-Grüneisen, multi-medium flow.

1 Introduction

The simulation of the interaction between compressible fluid and compressible solid is still a very challenging work, which is applied extensively in engineering research, such as vessel hull deformed by underwater blast waves, rock destroyed by underground explosions, high speed multi-medium impacts, and so on. In the above problems, not only

*Corresponding author. *Email addresses:* `cheney@pku.edu.cn` (L. Chen), `rli@math.pku.edu.cn` (R. Li) `yaocheng@pku.edu.cn` (C. B. Yao)

the densities and pressures vary largely across the material interface, but also the constitutive laws differ distinctly. Unlike fluids, solids can resist both the volumetric deformation and shear deformation simultaneously, and can undergo an elastoplastic deformation when suffering from strong impacts. Two types of elastoplastic models are mainly used to describe the finite deformation of solids, the hyper-elastoplastic model [1–5] and the hypo-elastoplastic model [6, 7]. Due to the simplicity of hypo-elastoplastic model, which is permitted to incorporate the plasticity effects through Maxwell-type relaxation models [5], it is extensively applied in engineering sciences, such as [7–15].

The simulation of the fluid-solid interaction problem mainly consists of two difficulties, the interface capture and the interaction between distinct mediums. Various techniques have been developed to overcome these difficulties, which can be classified into the Lagrangian method [16–18], Eulerian method [13–15, 19–23], ALE method [24, 25], and so on. The interface capture is accomplished distinctly due to the coordinates they choose [16], while the accuracy of the interaction greatly relies on the accurate prediction of the interface states. One common approach is to solve a multi-medium Riemann problem at the phase interface, which contains the fundamentally physical and mathematical properties of the governing equations and plays a key role in predicting the interface states. The solution of a multi-medium Riemann problem depends not only on the initial states at each side of the interface, but also on the forms of constitutive laws.

For multi-medium fluid flows, the exact solution of the Riemann problem for general convex equations of state has been investigated by [26–32]. However, the solution of the compressible fluid-compressible solid Riemann problem is more complex. Unlike the fluid, there may exist more than one nonlinear wave in a solid when it undergoes an elastoplastic deformation. The compatibility conditions across the interface between the fluid and solid, and the number of nonlinear waves in the solid will increase the difficulty to obtain the exact solution of the fluid-solid Riemann problem. Several methods have been studied to solve the above problems. Kaboudian *et al.* [33] analyzed the elastic Riemann problem in the Lagrangian framework, and established the corresponding Riemann solver according to the characteristic theory, where the constitutive equation was the Hooke's law for one-dimensional isotropic linearly elastic solid. Xiao *et al.* [34] raised an iterative procedure to solve the Riemann problem approximately by linearizing the Riemann invariants. Tang *et al.* [35] put forward a nearly exact Riemann solver for the hydro-elastoplastic solid based on the physical observation, where the Murnagham equation of state (EOS) and perfect plasticity model were chosen for the hydrostatic pressure and deviatoric stress respectively. Abouziarov *et al.* [36] and Bazhenov *et al.* [37] analyzed the structures of shock waves and rarefaction waves in an elastoplastic material on the assumption of barotropy, without taking into account the internal energy equation. Cheng *et al.* [17, 18] analyzed the wave structures of one-dimensional elastoplastic flows and developed a two-rarefaction approximate Riemann solver (TRRSE). Menshov *et al.* [38] provided an analysis of the Riemann problem in a complete statement for the perfect plasticity on the assumption of one-dimensional motion and uniaxial strain. Lin *et al.* [39] established a Riemann solver for the one-dimensional longitudinal and tor-