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An Improved Pressure-Equilibrium Diffuse Interface Model for Solid-Fluid Interaction

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Abstract. For solid-fluid interaction, one of the phasic density equations in diffuse interface models is degenerated to a "$0=0$" equation when the volume fraction of a certain phase takes the value of zero or unity, because the conservative variables in phasic density equations include volume fractions. The degeneracy can be avoided by adding an artificial quantity of another material into the pure phase. However, non-physical waves are introduced by the artificial treatment. In this paper, an improved pressure-equilibrium diffuse interface model, which is able to treat zero/unity volume fractions, is presented for solid-fluid interaction. In the proposed model, the phasic density equations are replaced by the algebraic relation between phasic densities and inverse deformation gradient tensors. In consequence, the volume fractions and the phasic densities do not appear explicitly in the conservative equations any more. The degeneracy introduced by zero/unity volume fractions are prevented. A flux-splitting based finite difference algorithm suitable for this formulation is then presented. A series of one-dimensional and two-dimensional numerical tests demonstrate that the proposed model can present more accurate results near material interfaces.

AMS subject classifications: 74F10

Key words: Solid-fluid interaction, diffuse interface model, phasic density equation, Mie-Grüneisen equation of state, Eulerian methods.

1 Introduction

Solid-fluid interaction with large deformations occur in various problems, such as penetration, explosion, and metal welding. Accurate capturing of interface between different materials is a big challenge in numerical simulation of such phenomena. Lagrangian framework [1] is widely used to simulate multi-material flows, as the interface can be computed naturally by Lagrangian methods with minimal grids. An alternative method

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is to simulate solids by Lagrangian methods and fluids by Eulerian methods [2,3], where solids and fluids are coupled by forces and velocities. However, these methods fails when treating problems which involve large deformations due to the mesh distortion [4]. A fully Eulerian method becomes more suitable for such situations. The most commonly used Eulerian framework for solid mechanics can be found in the works of Godunov [5] and Plohr [6], where the shear stress is controlled by the deformation gradient tensor or its inverse. In multi-material Eulerian computation, where the grid is fixed, the material interface may be inside a cell, leading to a mixture cell. An interface model should be considered to treat the mixture cell.

Two typical interface models existed in the literatures: the sharp interface model (SIM) and the diffuse interface model (DIM). Compared with SIM, such as volume of fluid method [7], level-set method [8–10], and immersed boundary method [11], diffuse interface model(DIM) [12] has been developed rapidly in recent years because it is simpler and more conservative. In DIM, flows in the whole domain are viewed as a mixture and the same equations are solved. The original seven equation models can be found in the works of Baer [13] and Saurel [14]. A series of reduced models were investigated by Saurel [15], Kapila [16] and Murrone [17], assuming pressure equilibrium or velocity equilibrium. Related high order numerical methods can be found in the works of Pan [18], Ghaisas [19], He [20,21] and Peng [22]. Direct extensions of Murrone's model and Saurel'model to solid-fluid interaction have been investigated by Favrie [23,24] and Ndanou [25]. However, it is found that some intrinsic deficiencies of DIM, which can be neglected for fluid-fluid interactions, appeared to take significant effect on solid-fluid interactions.

A well-known deficiency of most existing DIMs for solid-fluid interactions is that they cannot handle problems with pure phases. If the volume fraction of a certain material takes the value of zero or unity, one of the phasic density equations is degenerated to a "0=0" equation, because volume fractions are involved in conservative variables of phasic density equations. An artificial quantity of another material must be added into the pure phase to avoid the degeneracy of the model. This artificial treatment is reasonable in fluid-fluid interaction, because the physical properties of different fluids are similar. However, the physical properties and wave structures of solid and fluid are quite different. Even a small quantity of solids in gas phase may dramatically affect the physical structures. First, this small quantity of solids can generate non-zero shear stresses [24], and shear waves are introduced, which is unacceptable in pure gas. Second, the sound speeds of gas phases are dramatically altered by this small quantity of solids, which has been observed by Kluth [26] in a comparison of Lagrangian result and Eulerian result in an impact problem. So it is necessary to propose a model which is able to handle zero/unity volume fractions.

The difficulty in treating zero/unity volume fraction stems from the phasic density equations. To overcome it, several attempts have been made where mixture density equations were used instead of phasic density equations in fluid-fluid interactions. Abgrall [27] adopted this formulation and added a nonconservative Γ equation to describe