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An Implicit Scheme for Moving Walls and Multi-Material Interfaces in Weakly Compressible Materials

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Abstract. We propose a numerical method for the simulation of flows from weakly compressible to low Mach regimes in domains with moving boundaries. Non-miscible weakly compressible materials separated by an interface are included as well. The scheme is fully implicit and it exploits the relaxation all-speed scheme introduced in [1]. We consider media with significantly different physical properties and constitutive laws, as fluids and hyperelastic solids. The proposed numerical scheme is fully Eulerian and it is the same for all materials. We present numerical validations by simulating weakly compressible fluid/fluid, solid/solid and solid/fluid interactions.

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1 Introduction

Physical phenomena involving different materials arise in several applications such as multiphase flows, fluid-structure interaction and impacts. A possible simulation approach for these phenomena is the modelling of the material discontinuity with a diffused interface [2, 3]. This method is robust and prevents the formation of oscillations, thanks to an interface that corresponds to an artificial mixture of the two fluids. However, the interface amplitude is due to numerical diffusion, which increases with time evolution.

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The interface can also be considered as a contact discontinuity, especially in presence of interactions among different materials such as solid/solid and solid/fluid, or also between two non-miscible fluids. In this framework, the ghost fluid method has been introduced in [4] to model multiphase flows with sharp interfaces on Cartesian meshes. It consists in considering in every cell of the domain both the real fluid and the "ghost fluid", extrapolating from this latter some quantities at the interface. Other variants and extensions of the ghost fluid method have been proposed in several works [5–9]. Immersed boundary methods [10–12] are another option to keep the material interface sharp. With these methods, the contact discontinuity representing the interface can arbitrarily cross the grid and the transmission conditions are applied via interpolation. Several strategies have been proposed in literature to solve both moving boundaries and multi-material flows in the fully compressible regime, including Lagrangian models [13, 14] and Eulerian models [15–18].

In the present work we are concerned with the numerical simulation of multi-material flows in the weakly compressible regime and especially when the Mach number tends to zero. We address the solution of fluid-dynamics problems and also compressible solids deformations. The propagation of waves in heterogeneous compressible media can be affected by drastic changes in the speed of sound or in the speed of elastic waves. This is related to the different local stiffness of the considered materials. Thus, it is important to have a scheme that is able to deal with different regimes. Standard multi-material schemes are usually based on the explicit-upwind framework for the simulation of compressible flows. Such schemes may fail when approaching the low Mach number limit, since upwind discretizations provide an excessive numerical viscosity on the slow waves in this regime [19, 20]. The other drawback in adopting explicit methods for low Mach flows is related to the need of extremely small time steps, because in such schemes the CFL condition has to be imposed on the fastest wave speed to ensure stability. Due to this, the time step of compressible codes becomes extremely small as the incompressible regime is approached, increasing the computational time. In literature, several different schemes have been devised to deal with low Mach regimes, including implicit preconditioning methods [21–23] and semi-implicit asymptotic preserving schemes [24–26]. All these methods are specifically build to solve fluid flows.

We are able to overcome the problems mentioned above by coupling an implicit treatment of the interface with the all-speed scheme proposed in [1] for the simulation of compressible materials, including elastic solids. The scheme is general enough to deal with different materials, since it is based on a monolithic Eulerian model, describing each material (gas, liquid or solid) with the same system of conservation laws and an appropriate general formulation of the constitutive law [27–29]. The scheme is based on the Jin-Xin relaxation and it is fully implicit. In the present paper, we propose a method to impose boundary conditions on moving walls (piston problem) and a technique to solve flows across multi-material interfaces, even in the case when the interface velocity is unknown and determined by the flow. In particular, we are interested in an all-speed treatment, in order to accurately approximate moving boundaries and multi-material interfaces in