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Lagrangian Mesh Model with Regridding for Planar Poiseuille Flow

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Abstract. Many biological settings involve complex fluids that have non-Newtonian mechanical responses that arise from suspended microstructures. In contrast, Newtonian fluids are liquids or mixtures of a simple molecular structure that exhibit a linear relationship between the shear stress and the rate of deformation. In modeling complex fluids, the extra stress from the non-Newtonian contribution must be included in the governing equations.

In this study we compare Lagrangian mesh and Oldroyd-B formulations of fluid-structure interaction in an immersed boundary framework. The start-up phase of planar Poiseuille flow between two parallel plates is used as a test case for the fluid models. For Newtonian and Oldroyd-B fluids there exist analytical solutions which are used in the comparison of simulation and theoretical results. The Lagrangian mesh results are compared with Oldroyd-B using comparable parameters. A regridding algorithm is introduced for the Lagrangian mesh model. We show that the Lagrangian mesh model simulations with regridding produce results in close agreement with the Oldfoyd-B model.

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Key words: Lagrangian mesh model, Oldroyd-B, immersed boundary method, viscoelastic fluid, regridding methods.

1 Introduction

Complex fluids have become a major focus of attention in fluid mechanics. Many biological fluids have suspended microstructures and may exhibit complex, non-Newtonian responses. These include mucus in the lung, as well as fluids in the stomach, intestines,

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oviduct and uterus. A variety of analytical and numerical studies have emerged to investigate the mechanics of these fluids. The Oldroyd-B constitutive equations in an immersed boundary framework have been used to model complex fluids [5,8,12] and fluid-particle interactions [3, 10, 15, 20]. There are also several studies that use a Lagrangian mesh to model complex fluids [1,2,7,23,24].

An earlier numerical rheometer study [7] demonstrated that viscoelastic properties of an immersed boundary Lagrangian mesh model were very similar to those of an Oldroyd-B model [7]. Here we compare the two models in start-up Poiseuille flow. Planar Poiseuille flow between two parallel plates, also known as parabolic channel flow, driven by a constant spatially-uniform pressure gradient, is a well-known test problem for numerical algorithms. It is useful as a benchmark problem because of the existence of analytical solutions for both Newtonian and Oldroyd-B viscoelastic flow, thus allowing exact tracking of discretization and lagging-errors [9]. Waters and King (1970) studied the time-dependent start-up flow of viscoelastic fluids and found an analytical solution for planar Poiseuille flow. This solution can be compared with numerical simulation results using various model parameters, such as elasticity and Weissenberg number (W_i). The order of convergence of discretization errors can be established by successive refinement of the fluid grid, Lagrangian mesh. An important finding during the present course of study is that refinement of fluid grid or Lagrangian mesh, as well as remeshing of the Lagrangian mesh can produce velocity profiles that are very similar to the analytical solutions from the corresponding Oldroyd-B model.

2 Oldroyd-B model

The Oldroyd-B formulation [16] is frequently used to model complex fluids. Compared with Newtonian fluid, viscoelastic fluid can have a dilute suspension of high molecular weight polymer structures in a Newtonian solvent (water, glycerol, etc.). Distended polymers provide an extra stress to the solvent stress through random walks caused by collisions with solvent molecules. The Oldroyd-B (OB) model incorporates this additional stress by modeling this component separately and adding it to the total stress. An immersed boundary Oldroyd-B (IB-OB) method was proposed by [18] for Stokesian peristaltic pumping.

We shall consider the start-up flow of viscoelastic fluids in a planar channel. The flow is driven by an instantaneously applied uniformly distributed pressure gradient to the fluid initially at rest. The channel is bounded by two parallel plates with a separation distance $2H$. The channel flow has x-periodic geometry as illustrated in Fig. 1. An IB-OB model for 2D Poiseuille flow is formulated as follows. With the assumptions that the fluid is isothermal and incompressible, the fundamental conservation equations of linear momentum and mass within the fluid domain Ω are given by

$$\rho\left(\frac{D\mathbf{u}}{Dt}\right)=\nabla\cdot\mathbf{S}_{tot} \quad \text{and} \quad \nabla\cdot\mathbf{u}=0 \quad \text{in}\,\Omega, \tag{2.1}$$