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Global Existence and Blow-up for a Two-dimensional Attraction-repulsion Chemotaxis System

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Abstract: This paper is devoted to dealing with the parabolic-elliptic-elliptic attraction-repulsion chemotaxis system. We aim to understand the competition among the repulsion, the attraction, the nonlinear productions and give conditions of global existence and blow-up for the two-dimensional attraction-repulsion chemotaxis system.
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1 Introduction

Chemotaxis is a phenomenon which describes the movement of cells in response to the concentration gradient of the chemical produced by cells themselves. The famous chemotaxis model was first proposed by Keller and Segel[1] in 1970. The Keller-Segel model can be read as follows:

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \nabla\cdot(\chi u\nabla v) + f(u), & x\in\Omega,\ t\in(0,\,T),\\ \tau v_t = \Delta v + u - v, & x\in\Omega,\ t\in(0,\,T),\end{cases} \tag{1.1}$$

where $u = u(x,\,t)$ denotes the density of cells, $v = v(x,\,t)$ represents the concentration of the chemoattractant. The function $f : [0,\,\infty) \longrightarrow \mathbf{R}$ is smooth, χ is the parameter referred as chemosensitivity. The system (1.1) with $\tau = 0$ or $\tau = 1$ has been studied extensively in the past four decades. For instance, when $D(u) \equiv 1$, $\tau = 0$ and $\Omega \subset \mathbf{R}^2$ is a bounded

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domain, the solution to (1.1) is global bounded provided that $f(u) = \mu u(1-u)$ with $\mu > 0$ (see [2]).

In order to better understand the parabolic-elliptic-elliptic attraction-repulsion chemotaxis system, let us mention previous contributions as follows:

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v), & x\in\Omega,\ t>0, \\ 0 = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0. \end{cases} \tag{1.2}$$

When $n = 1$, the main results in [3] showed that the (1.2) admits a unique global solution. Nagai[4] found that there exists the critical mass $m_c = \dfrac{8\pi}{\chi\alpha}$ which determines the behavior of the solution when $n = 2$. Precisely, if the initial mass $\displaystyle\int_\Omega u_0(x)\mathrm{d}x \le m_c$, the solution of system (1.2) is global and bounded, whereas the finite time blow-up happens when $\displaystyle\int_\Omega u_0(x)\mathrm{d}x > m_c$. In addition, the blow-up may occur when $\displaystyle\int_\Omega u_0(x)\mathrm{d}x > \dfrac{4\pi}{\chi\alpha}$ in some special Ω (see [5]–[8]). When $n \ge 3$, Winker[9] showed that there exists radially symmetric solution blowing up in finite time with proper initial conditions.

In numerous biological processes, general mechanisms in cell include not only attractive but also repulsive signals, which can form various interesting biological patterns (see [10]). Then the model can be expressed as following attraction-repulsion chemotaxis system:

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w), & x\in\Omega,\ t\in(0,T), \\ \tau v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t\in(0,T), \\ \tau w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t\in(0,T). \end{cases} \tag{1.3}$$

The system (1.3) is proposed by [11] to describe the aggregation of microglia observed in Alzheimer's disease. Fewer blow-up results are available for system (1.3) than (1.1), because (1.3) relies on Lyapunov function. When $n = 1$, $\tau = 1$, global existence, non-trivial stationary, asymptotic behavior and pattern formation of solutions to the system (1.3) have been studied in [12]–[13]; when $n = 2$, $\tau = 1$, if $\beta \ne \delta$ and repulsion prevails over attraction (i.e., $\xi\gamma - \chi\alpha > 0$), then the system (1.3) admits a unique global bounded solution (see [14]); in the case $\tau \equiv 0$, Yu *et al.*[15] proved that the finite time blow-up for the nonradial solution happens when

$$\chi\alpha - \xi\gamma > 0, \qquad \int_\Omega u_0(x)\mathrm{d}x > \frac{8\pi}{\chi\alpha - \xi\gamma}$$

and $\displaystyle\int_\Omega u_0(x)|x-x_0|^2\mathrm{d}x$ sufficiently small for some $x_0 \in \Omega$; Li and Li[16] showed that the finite time blow-up for the nonradial solution of (1.3) happens under either the condition

$$\chi\alpha - \xi\gamma > 0, \qquad \int_\Omega u_0(x)\mathrm{d}x > \frac{8\pi}{\chi\alpha - \xi\gamma}$$

when $\delta \ge \beta$, or

$$\chi\alpha\delta - \xi\gamma\beta > 0, \qquad \int_\Omega u_0(x)\mathrm{d}x > \frac{8\pi}{\chi\alpha\delta - \xi\gamma\beta}$$

if $\delta < \beta$. For more detail results on attraction-repulsion chemotaxis system, we refer the readers to [17]–[20]. Blow-up is an extremely behavior. In order to restrain the behavior, people add the logistic source. More detail results on attraction-repulsion chemotaxis system with the logistic source, the readers can see [21]–[24].