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Global and Bifurcation Analysis of an HIV Pathogenesis Model with Saturated Reverse Function

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Abstract: In this paper, an HIV dynamics model with the proliferation of CD4 T cells is proposed. The authors consider nonnegativity, boundedness, global asymptotic stability of the solutions and bifurcation properties of the steady states. It is proved that the virus is cleared from the host under some conditions if the basic reproduction number R_0 is less than unity. Meanwhile, the model exhibits the phenomenon of backward bifurcation. We also obtain one equilibrium is semi-stable by using center manifold theory. It is proved that the endemic equilibrium is globally asymptotically stable under some conditions if R_0 is greater than unity. It also is proved that the model undergoes Hopf bifurcation from the endemic equilibrium under some conditions. It is novelty that the model exhibits two famous bifurcations, backward bifurcation and Hopf bifurcation. The model is extended to incorporate the specific Cytotoxic T Lymphocytes (CTLs) immune response. Stabilities of equilibria and Hopf bifurcation are considered accordingly. In addition, some numerical simulations for justifying the theoretical analysis results are also given in paper.

Key words: HIV model; global asymptotical stability; center manifold theory; Hopf bifurcation; backward bifurcation

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1 Introduction

Human Immunodeficiency Virus (HIV), which destroys CD4 T cells and decreases the resistance of the immune system. The count of CD4 T cells is a primary indicator used to measure progression of HIV infection. In a normal healthy individual's blood, the level of CD4 T cells is between 800 and 1200 mm^{-3} (see [1]–[3]). When CD4 T cells count of one patient reaches 200 mm^{-3} or below, the person is called AIDS patient. The amount of virus rises dramatically after primary infection and falls to a lower level after a few weeks to months. At first, it is estimated that as many as 10^{10} virions are produced and destroyed in an infected individual each day. The amount of virus rises again after ten years or so (see [1]). By utilizing mathematical models, we can better understand HIV dynamics, disease progress and interaction of HIV and the immune system. Many ordinary differential equations for HIV infection pathogenesis have been proposed and investigated by bio-mathematicians (see [1] and [3]–[5]). The basic mathematical model which describes HIV infection dynamics has been studied in [1] and [4]. Global stabilities have been established by using Lyapunov functions. Many papers consider the proliferation of the CD4 cells (see [3] and [6]). Motivated by the above works, we propose one HIV model which is described by the following system of differential equations

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = s - \delta T(t) - \beta T(t)v(t) + rT(t)\frac{v(t)}{k+v(t)}, \tag{1.1}$$

$$\frac{\mathrm{d}T^*(t)}{\mathrm{d}t} = \beta T(t)v(t) - aT^*(t), \tag{1.2}$$

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = bT^*(t) - cv(t). \tag{1.3}$$

State variables $T(t)$, $T^*(t)$ and $v(t)$ represent the concentration of uninfected CD4 T cells, infected CD4 T cells, and the HIV particles in the blood, respectively. The human body produces CD4 T cells at a constant rate s. The parameters δ, a and c denote the death rate of uninfected CD4 T cells, infected CD4 T cells and the virus particles, respectively. Here we use bilinear incident rate $\beta T(t)v(t)$ to describe the infection incident rate. The term $\dfrac{rT(t)v(t)}{k+v(t)}$ denotes saturated reverse function response, the proliferation of CD4 T cells, where r is the maximal proliferation rate, k is half-saturation constant of the proliferation process and can be considered as michaelis-menten constant. Free HIV are produced from actively infected cells at rate $bT^*(t)$ and are removed at rate $cv(t)$.

The main purpose of this paper is to analyse global properties for the system (1.1)–(1.3) and explore the impact of saturated reverse function response on the dynamical behavior of the system. We begin model analysis with proving the positivity and boundedness of the solutions of the system. We prove that system (1.1)–(1.3) exhibits the backward bifurcation and Hopf bifurcation under some conditions. We prove that the endemic equilibrium is globally asymptotically stable under some conditions by using the geometric approach, developed by Li and Muldowney (see [2], [7]–[8]). We also research the effect of CTLs on HIV infection and give the conditions of Hopf bifurcation.