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An SIRS Epidemic Model with Pulse Vaccination, Birth Pulse and Logistic Death Rate

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Abstract: In this paper, we propose an SIRS epidemic model with pulse vaccination, birth pulse and Logistic death rate. By using the stroboscopic map of a discrete dynamical system, the disease-free periodic solution (DFPS for short) of the model under pulse vaccination and birth pulse is obtained. Based on the Floquet theory and comparison theorem of impulsive differential equations, the global asymptotic stability of the DFPS is given, and sufficient conditions for the permanence of the model are obtained. In addition, numerical simulations are done to confirm our theoretical results.

Key words: Logistic death rate, birth pulse, threshold value, global stability, permanence

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1 Introduction

Infectious diseases have tremendous influence on human life. Every year, millions of people died of various infectious diseases. Controlling infectious diseases have been an increasingly complex issue in recent year. Pulse vaccination has been testified to be an effective strategy in preventing such viral infectious as rabies, yellow fever, polio virus, encephalitis B. For many epidemic models, we always assume that the population birth rate is either a constant or proportional to the number of population. In short, it is assumed that the population is

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always born at all times. Generally speaking, this assumption is not reasonable for those large number of individuals and population with overlapping generations. For some animals, especially some sea animals, these animals usually produce offspring in the special months of year. In other words, the birth rate is not a continuous function of time, but appears in the form of pulse. In addition, in order to control the epidemic of infectious diseases, some vaccines are often carried out in a certain time every year, which is also a form of pulse. On the basis of facts, Ma *et al.*[1] discussed a model with birth pulse, and assumed that the birth rate is constant. In [2]–[5], the authors hypothesized that the birth rate and the mortality rate are density dependent and assumed that population is born in the form of pulse. On the basis of those assumptions, the conditions of equilibrium points stability were given. In [6]–[11], the authors analyzed that pulse vaccination has influence on control of infectious disease. Han and Wei[12] studied a delay SEIR epidemic model with birth pulse, pulse vaccination and vertical infection, obtained two thresholds and analyzed the stability of the model. In [13] and [14], the authors discussed the branch questions of the model with birth pulse and pulse vaccination. In [12], the authors vaccinated all newborns. Here, only pulse vaccination is given to susceptible people in this paper. We establish that pulse vaccination and birth pulse occur at the same time and consider vertical infection. This is different from [13] and [14]. In short, in this paper, based on the previous articles [12]–[14], we consider the birth pulse and pulse vaccination at the same time, and we assume that the disease has vertical infection and mortality due to disease, we establish compartment model and analyze existence of the DFPS, we further discuss stability and permanence of disease.

The organization of this paper is as follows: In Section 1, we establish an SIRS epidemic model. In Section 2, by using the stroboscopic map of a discrete dynamical system, we analyze existence of the DFPS and deal with the global attractivity of the DFPS. In Section 3, we discuss the permanence of model. In Section 4, we do numerical simulations to confirm our theoretical results. Finally, we make a summary for this article.

2 Model Description

In order to describe the model, $S(t)$, $I(t)$ and $R(t)$ represent the number of the susceptible, the infected and the recovered at time t, respectively. The total population number at time t is expressed by

$$N(t) = S(t) + I(t) + R(t).$$

Main assumptions are as follows:

(i) $\mu N(t)$ is exponential immigrant for the susceptible.

(ii) Adopting Logistic death rate $d + \dfrac{\mu N(t)}{K}$ (see [15]–[17]), d is the natural death rate and K is the environmental capacity.

(iii) Adopting standard incidence rate $\dfrac{\beta S(t)I(t)}{N(t)}$.