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Hypersemilattice Strongly Regular Relations on Ordered Semihypergroups

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Abstract: In this paper, we first consider the regular and strongly regular relations on ordered semihypergroups in detail. In particular, we introduce the concepts of the hypersemilattice strongly regular relations and complete hypersemilattice strongly regular relations on ordered semihypergroups, and investigate their related properties. Furthermore, the properties of hyperfilters of an ordered semihypergroup are studied, and several related applications are given. Especially, we prove that the equivalence relation $\mathcal{N}$ on an ordered semihypergroup S is the least complete hypersemilattice strongly regular relation on S.

Key words: ordered semihypergroup, strongly regular relation, hyperfilter

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1 Introduction

As we know, an ordered semigroup $(S, \cdot, \leq)$ is a semigroup $(S, \cdot)$ with an order relation " $\leq$ " such that $a \leq b$ implies $xa \leq xb$ and $ax \leq bx$ for any $x \in S$. Ordered semigroups have several applications in the theory of sequential machines, formal languages, computer arithmetics and error-correcting codes. Similar to the theory of congruences on semigroups, congruences on ordered semigroups play an important role in studying the structures of ordered semigroups. For any congruence ρ on an ordered semigroup S, in general, we

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do not know whether the quotient semigroup S/ρ is also an ordered semigroup. Even if S/ρ is an ordered semigroup, the order on S/ρ is not necessarily relative to the order on the original ordered semigroup S. As to the above-mentioned questions, Kehayopulu and Tsingelis[1]–[2] introduced the concept of pseudoorder on an ordered semigroup S and proved that if σ is a pseudoorder on S, then there exists a congruence $\bar{\sigma}$ on S such that $S/\bar{\sigma}$ is an ordered semigroup. Since then, Xie and Wu[3] introduced the concept of regular semilattice congruences on an ordered semigroup S, and proved that the semilattice congruences $\mathcal{N}$ on S is the least regular semilattice congruence but not the least semilattice congruence on S.

The hyperstructure theory was introduced in 1934 by Marty[4] at the 8th congress of Scandinavian Mathematicians. Nowadays, hyperstructures have a lot of applications in several domains of mathematics and computer science, for example, see [5] and [6]. In the hyperstructure theory, semihypergroups are the simplest algebraic hyperstructures which are a generalization of concept of semigroups. Recently, a theory of hyperstructures on ordered semigroups has been developed. In [7], Heidari and Davvaz applied the theory of hyperstructures to the ordered semigroups and introduced the concept of the ordered semihypergroups, which is a generalization of the concept of the ordered semigroups. Later on, a lot of papers on ordered semihypergroups have been written, for instance, see [8]–[12]. It is worth pointing out that Davvaz *et al.*[8] introduced the concepts of the regular and strongly regular relations on ordered semihypergroups, and extended some results in [1] on ordered semigroups to ordered semihypergroups. As a further study of (strongly) regular relations on ordered semihypergroups, in this paper we define and study the hypersemilattice strongly regular relations and complete hypersemilattice strongly regular relations on ordered semihypergroups, and extend some results in ordered semigroups to ordered semihypergroups. Furthermore, we study the properties of hyperfilters of an ordered semihypergroup in detail, and give several applications of hyperfilters in ordered semihypergroups. Especially, we consider the equivalence relation "$\mathcal{N}$" on an ordered semihypergroup, which is defined by $a\mathcal{N}b$ if and only if $N(a) = N(b)$, where $N(x)$ is the hyperfilter of S generated by x ($x \in S$). We prove that $\mathcal{N}$ is the least complete hypersemilattice strongly regular relation on S. As an application of the results of this paper, the corresponding results of semihypergroups (without order) and ordered semigroups are also obtained by moderate modifications.

2 Preliminaries and Some Notations

Recall that a hypergroupoid $(S, \circ)$ is a nonempty set S together with a hyperoperation, that is a map $\circ\colon S \times S \to P^*(S)$, where $P^*(S)$ denotes the set of all the nonempty subsets of S. The image of the pair (x, y) is denoted by $x \circ y$. If $x \in S$ and A, B are nonempty subsets of S, then $A \circ B$ is defined by $A \circ B = \bigcup\limits_{a\in A,\, b\in B} a \circ b$. Also $A \circ x$ is used for $A \circ \{x\}$ and $x \circ A$ for $\{x\} \circ A$. Generally, the singleton $\{x\}$ is identified by its element x.

We say that a hypergroupoid $(S, \circ)$ is a semihypergroup if the hyperoperation "'$\circ$" is