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Some Topological Properties of Charming Spaces

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Abstract: In this paper, we mainly discuss the class of charming spaces. First, we show that there exists a charming space such that the Tychonoff product is not a charming space. Then we discuss some properties of charming spaces and give some characterizations of some class of charming spaces. Finally, we show that the Suslin number of an arbitrary charming rectifiable space is countable.

Key words: charming space, (i, j)-structured space, Lindelöf Σ-space, Suslin number, rectifiable space

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1 Introduction

In 1969, Nagami[1] introduced the notion of Σ-spaces, and then the class of Σ-spaces with the Lindelöf property (i.e., the class of Lindelöf Σ-space) quickly attracted the attention of some topologists. From then on, the study of Lindelöf Σ-spaces has become an important part in the functional analysis, topological algebra and descriptive set theory. Tkachuk[2] described detailedly Lindelöf Σ-spaces and made an overview of the recent progress achieved in the study of Lindelöf Σ-spaces. Arhangel'skii[3] has proved if the weight of X does not exceed 2^ω, then any remainders of X in a Hausdorff compactification is a Lindelöf Σ-space.

It is natural to ask if we can find a class of spaces $\mathscr{P}$ such that each remainder of a Hausdorff compactification of arbitrary metrizable space belongs to $\mathscr{P}$. Therefore, Arhangel'skii

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defined charming spaces in [3] and showed the any remainder of paracompact p-space in a Hausdorff compactification is a charming space. Indeed, Arhangel'skii defined many new classes of spaces (that is, (i, j)-structured spaces) which have similar structure with the class of charming spaces, and he said that each of the classes of spaces so defined is worth studying. Therefore, we mainly discuss some topological properties of (i, j)-structured spaces.

2 Preliminaries

All spaces are Tychonoff unless stated otherwise. Readers may refer to [4]–[5] for notations and terminology not explicitly given here.

Definition 2.1 *Let $\mathscr{N}$ be a family of subsets of a space X. Then the family $\mathscr{N}$ is a network of X if every open subset U is the union of some subfamily of $\mathscr{N}$.*

Definition 2.2 *We say that a space is cosmic if it has a countable network.*

Definition 2.3 *Let X be a space. We say that X is a Lindelöf p-space if it is the preimage of a separable metrizable space under a perfect mapping.*

Definition 2.4 *X is a Lindelöf Σ-space if there exists a space Y which maps continuously onto X and perfectly onto a second countable space.*

That is, a Lindelöf Σ-space is the continuous onto image of some Lindelöf p-space. Therefore, a Lindelöf p-space is a Lindelöf Σ-space. It is well-known that the class of Lindelöf Σ-spaces contains the classes of Σ-compact spaces and spaces with a countable network.

Definition 2.5[3] *Let X be a space. If there exists a Lindelöf Σ-subspace Y such that for each open neighborhood U of Y in X we have $X \setminus U$ is also a Lindelöf Σ-subspace, then we say that X is a charming space.*

Definition 2.6[3] *Let $\mathscr{P}$ and $\mathscr{Q}$ be two classes of topological spaces respectively. A space X will be called $(\mathscr{P}, \mathscr{Q})$-structured if there is a subspace Y of X such that $Y \in \mathscr{P}$, and for each open neighborhood U of Y in X, the subspace $X \setminus U$ of X belongs to $\mathscr{Q}$. In this situation, we call Y a $(\mathscr{P}, \mathscr{Q})$-shell of the space X.*

Definition 2.7[3] *Let $\mathscr{P}_0$ be the class of Σ-compact spaces, $\mathscr{P}_1$ be the class of separable metrizable spaces, $\mathscr{P}_2$ be the class of spaces with a countable network, $\mathscr{P}_3$ be the class of Lindelöf p-spaces, $\mathscr{P}_4$ be the class of Lindelöf Σ-spaces and $\mathscr{P}_5$ be the class of compact spaces. Choose some $i, j \in \{0, 1, 2, 3, 4, 5\}$. A space X is called (i, j)-structured if it is $(\mathscr{P}_i, \mathscr{P}_j)$-structured. In particular, a $(4, 4)$-structured space is called a charming space.*