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Second Hankel Determinants and Fekete-Szegö Inequalities for Some Sub-Classes of Bi-Univalent Functions with Respect to Symmetric and Conjugate Points Related to a Shell Shaped Region

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Abstract. In this paper, we have investigated second Hankel determinants and Fekete-Szegö inequalities for some subclasses of Bi-univalent functions with respect to symmetric and Conjugate points which are subordinate to a shell shaped region in the open unit disc Δ.

Key Words: Analytic functions, univalent functions, Bi-univalent functions, second Hankel determinants, Fekete-Szegö inequalities, symmetric points, conjugate points.

AMS Subject Classifications: 30C45, 30C50, 30C80

1 Introduction

Let A be the class of all functions of the form

$$f(z)=z+\sum_{k=2}^{\infty}a_k z^k, \tag{1.1}$$

which are analytic in the open unit disc $\Delta=\{z:|z|<1\}$. Let S be the class of all functions in A which are univalent in Δ.

Let P denote the family of functions $p(z)$ which are analytic in Δ such that $p(0)=1$, and $\Re p(z)>0$ $(z\in\Delta)$ of the form $P(z)=1+\sum_{n=1}^{\infty}c_n z^n$.

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For two functions f and$g$, analytic in Δ, we say that the function f is subordinate to g in Δ and we write it as $f(z) \prec g(z)$ if there exists a Schwartz function ω, which is analytic inΔ with $\omega(0)=0$, $|\omega(z)| \leq 1$ $(z \in \Delta)$ such that

$$f(z)=g(\omega(z)). \tag{1.2}$$

Indeed, it is known that $f(z) \prec g(z) \Rightarrow f(0)=g(0)$ and $f(\Delta) \subset g(\Delta)$.

In 1959, Sakaguchi [26] defined a subclass S_s^* of S which satisfies following condition

$$\operatorname{Re}\left(\frac{2zf'(z)}{f(z)-f(-z)}\right)>0, \quad z \in \Delta.$$

The functions in the class S_s^* are starlike with respect to symmetric points. Further Sakaguchi has shown that the functions in S_s^* are close-to-convex and hence are univalent. The concept of starlike functions with respect to symmetric points have been extended to starlike functions with respect to N-symmetric points by Ratanchand [24] and Prithvipal Singh [21], Ram Reddy [22] studied the class of closeto-convex functions with respect to N-symmetric points and proved that the class is closed under convolution with convex univalent functions. Das and Singh [3] introduced another class C_s namely convex functions with respect to symmetric points and satisfying the condition

$$\operatorname{Re}\left(\frac{2(zf'(z))'}{(f(z)-f(-z))'}\right)>0, \quad z \in \Delta.$$

From the definition of S_s^* and C_s it is evident that $f \in C_s$ if and only if $zf(z) \in S_s^*$. Ashwah and Thomas in [6] introduced another class namely the class S_c^* consisting of functions starlike with respect to conjugate points.

Let S_c^* be the subclass of S consisting of functions given by (1.1) and satisfying the condition

$$\operatorname{Re}\left(\frac{2zf'(z)}{f(z)+\overline{f(\overline{z})}}\right)>0, \quad z \in \Delta.$$

In terms of subordination following Ma and Minda, Ravichandran [25] defined the classes $S_s^*(\phi)$ and $C_s(\phi)$ as below.

A function $f \in A$ is in the class $S_s^*(\phi)$ if

$$\frac{2zf'(z)}{f(z)-f(-z)} \prec \phi(z), \quad z \in \Delta.$$

And in the class $C_s(\phi)$ if

$$\frac{2(zf'(z))'}{(f(z)-f(-z))'} \prec \phi(z), \quad z \in \Delta.$$