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Hopf Bifurcation of a Nonresident Computer Virus Model with Delay

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Abstract. In this paper, a delayed nonresident computer virus model with graded infection rate is considered in which the following assumption is imposed: latent computers have lower infection ability than infectious computers. With the aid of the bifurcation theory, sufficient conditions for stability of the infected equilibrium of the model and existence of the Hopf bifurcation are established. In particular, explicit formulae which determine direction and stability of the Hopf bifurcation are derived by means of the normal form theory and the center manifold reduction for functional differential equations. Finally, a numerical example is given in order to show the feasibility of the obtained theoretical findings.

Key Words: Computer virus, delay, Hopf bifurcation, SLA model, Periodic solution.

AMS Subject Classifications: 34C15, 34C23, 37G15, 37N25

1 Introduction

With the advance of software and hardware technologies, computer viruses have been a major threat to our daily life [1]. It is an important matter to understand the spread law of computer viruses over the network. To achieve this goal, many dynamical models, such as SIR model [2], SIRS model [3–5], SEIR model [6], SEIRS model [7,8] and SEIQRS model [9,10] have been established by scholars at home and abroad to characterize the propagation of computer viruses.

Recently, the nonresident computer viruses that do not store or execute themselves from the computer memory have caused the attentions of many researchers [11]. In order

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to analyze and protect against the nonresident computer viruses, Muroya and Kuniya proposed the following SLA computer virus model [12]:

$$\begin{cases} \dfrac{dS(t)}{dt}=b-\mu_1S(t)-\beta_1S(t)L(t)-\beta_2S(t)A(t)+\gamma_1L(t)+\gamma_2A(t), \\ \dfrac{dL(t)}{dt}=\beta_1S(t)L(t)+\beta_2S(t)A(t)+\alpha_2A(t)-(\mu_2+\alpha_1+\gamma_1)L(t), \\ \dfrac{dA(t)}{dt}=\alpha_1L(t)-(\mu_3+\alpha_2+\gamma_2)A(t), \end{cases} \tag{1.1}$$

where $S(t)$, $L(t)$ and $A(t)$ denote the numbers of uninfected computers, latent computers and infectious computers at time t, respectively; b is the number of external computers that are accessed to the network at time t; μ_1, μ_2 and μ_3 are the rates at which the uninfected computers, latent computers and infectious computers are disconnected from the network; α_1 and α_2 are the rates of the nonresident computer viruses within latent computers are loaded into memory and nonresident computer viruses within infectious computers transfer control to the application program, respectively; β_1 and β_2 are the transmission rates of latent computers and infectious computers, respectively; γ_1 and γ_2 are the cure rates of latent computers and infectious computers, respectively. All the parameters in system (1.1) are positive constant. Muroya and Kuniya [12] investigated global stability and permanence of system (1.1).

However, studies on dynamical systems not only involve stability and permanence, but also involve some others such as bifurcation phenomenon and periodic solutions. Particularly, Hopf bifurcation of the dynamical systems with time delay are of considerable interest [13–16]. Motivated by the work above and considering that the nonresident computer viruses within latent computers need a period to be loaded into memory, we consider the following system with time delay:

$$\begin{cases} \dfrac{dS(t)}{dt}=b-\mu_1S(t)-\beta_1S(t)L(t)-\beta_2S(t)A(t)+\gamma_1L(t)+\gamma_2A(t), \\ \dfrac{dL(t)}{dt}=\beta_1S(t)L(t)+\beta_2S(t)A(t)+\alpha_2A(t)-(\mu_2+\gamma_1)L(t)-\alpha_1L(t-\tau), \\ \dfrac{dA(t)}{dt}=\alpha_1L(t-\tau)-(\mu_3+\alpha_2+\gamma_2)A(t), \end{cases} \tag{1.2}$$

where τ_1 is the time delay due to the period that the nonresident computer viruses within latent computers need to be loaded into memory.

The subsequent materials of this paper are organized as follows. In Section 2, stability of the infected equilibrium and existence of Hopf bifurcation are discussed by analyzing the characteristic equation of system (1.2). The formulas for determining the properties of the Hopf bifurcation are derived by using the normal form method and center manifold theory. Then, a numerical example is carried out to illustrate the validity of the theoretical results. Finally, conclusions are given in the last section.