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Approximation for Certain Stancu Type Summation Integral Operator

Prerna Maheshwari*

Department of Mathematics, SRM University Delhi-NCR Campus, Modinagar (UP), India

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Abstract. In the present paper, we consider Stancu type generalization of the summation integral type operators discussed in [15]. We apply hypergeometric series for obtaining moments of these operators. We also discuss about asymptotic formula and error estimation in terms of modules of continuity.

Key Words: Linear positive operators, hypergeometric series, modulus of continuity.

AMS Subject Classifications: 41A25, 41A28

1 Introduction

H. M. Srivastava and V. Gupta [15], proposed a certain family of linear positive operators defined as

$$G_n(f,x)=n\sum_{k=1}^{\infty}p_{n,k}(x,c)\int_0^{\infty}p_{n+c,k-1}(t)f(t)dt+p_{n,0}(x,c)f(0), \tag{1.1}$$

$x\in[0,\infty)$, where

$$p_{n,k}(x,c)=(-1)^k\frac{x^k}{k!}\phi_{n,c}^{(k)}(x) \tag{1.2}$$

and

- for $c=0$, $\phi_n(x)=e^{-nx}$, we obtain Phillips operators,
- for $c\in N$, $\phi_n(x)=(1+cx)^{\frac{-n}{c}}$, we get the discretely defined Baskakov-Durrmeyer operators.

The sequence $\{\phi_n\}_{n\in N}$ of the function defined on an interval $[0,b]$, $b>0$ satisfies the following properties for every $n\in N$, $k\in N_0$,

*Corresponding author. *Email address:* `mprerna_anand@yahoo.com` (P. Maheshwari)

1. $\phi_{n,c} \in C^\infty([a,b])$,
2. $\phi_{n,c}(0)=1$,
3. $\phi_{n,c}$ is completely monotone i.e., $(-1)^k \phi_{n,c}^{(k)} \geq 0$,
4. There exist an integer c such that $\phi_{n,c}^{(k+1)}(x) = -n\phi_{n+c,c}^{(k)}(x)$, $n > \max\{0,c\}$.

Remark 1.1. Functions $\phi_{n,c}$ have many applications in different fields of Science and Mathematics like potential theory, probability theory, Physics and Numerical Analysis. A collection of most interesting properties of such functions can be found in [17].

These operators are also termed as Srivastava-Gupta operators (see [2,10,16]). In [7], authors have considered the Bezier variant of these operators and estimated the rate of convergence for functions of bounded variation. Motivated by the sequence G_n, Gupta et al. [4] also defined a mixed family of summation integral operators with different weight function. In approximation theory the genuine type of operators are very important, as they are defined implicitly with values of functions at end points of the interval in which the operators are defined. In 1954, Phillips [14] introduced such operators and later Mazhar and Totik [8] discussed these operators in different form.

In [5, 11, 12] authors have also studied in this direction and discussed different approximation properties of various operators.

Based on two parameters α, β and satisfying the condition $0 \leq \alpha \leq \beta$, motivated by the recent work on Stancu type of generalization (see [1, 9, 13]) in the present paper, we consider the Stancu type generalization of operators (1.1) as

$$G_{n,c}^{\alpha,\beta}(f,x) = n\sum_{v=1}^{\infty} p_{n,v}(x,c)\int_0^\infty p_{n+c,v-1}(t,c) f\left(\frac{nt+\alpha}{n+\beta}\right) dt + p_{n,0}(x,c) f\left(\frac{\alpha}{n+\beta}\right), \tag{1.3}$$

where $p_{n,v}(x,c)$ is defined above in (1.2). In this paper, we study simultaneous approximation for the case $c=1$ of the operators defined in (1.3) and establish Voronovskaja type asymptotic formula and error estimation. To obtain moments by using hypergeometric series, we use the technique developed by [6].

2 Alternate forms

The operators $G_{n,c}^{\alpha,\beta}(f,x)$ for the case $c=1$ can be written as below. For $x \in [0,\infty)$

$$G_{n,1}^{\alpha,\beta}(f,x) = \int_0^\infty K_n(x,t) f\left(\frac{nt+\alpha}{n+\beta}\right) dt, \tag{2.1a}$$

$$\begin{aligned} G_{n,1}^{\alpha,\beta}(f,x) = &\sum_{v=1}^{\infty} p_{n,v}(x)\int_0^\infty b_{n,v-1}(t) f\left(\frac{nt+\alpha}{n+\beta}\right) dt \\ &+ (1+x)^{-n} f\left(\frac{\alpha}{n+\beta}\right), \quad x \in [0,\infty), \end{aligned} \tag{2.1b}$$