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The Meromorphic Solutions of the Zakharov-Kuznetsov Modified Equal Width Equation

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Abstract. In this paper, we use the complex method to obtain all meromorphic solutions of the complex Zakharov-Kuznetsov modified equal width equation, then find the exact traveling wave solutions of the Zakharov-Kuznetsov modified equal width equation. At last, we give some computer simulations to illustrate our main results.

Key Words: Exact solution, meromorphic function, elliptic function.

AMS Subject Classifications: 30D35, 34A05

1 Introduction

One of the main topics of mathematical physics is to find exact solutions of nonlinear partial differential equations. These equations play an important role in fluid mechanics, plasma physics, optical fibres, solid-state physics and chemical physics and other fields. In 2009, A. Biswas [1, 2] obtained the solitary wave solutions, topological solitons and non-topological solitons of the generalized Zakharov-Kuznetsov modified equal width equation (1.1) by using the solitary wave ansatz method. In 2014, Y. Pandir [3] obtained the soliton solutions, rational solutions and elliptic integral function solutions of the generalized Zakharov-Kuznetsov modified equal width equation by using the extended trial equation method. In this paper, we use the complex method to obtain all meromorphic solutions of the Zakharov-Kuznetsov modified equal width (ZK-MEK) equation.

The Zakharov-kuznetsov modified equal width equation [1–3] is

$$u_t+a(u^3)_x+(bu_{xt}+cu_{yy})_x=0, \tag{1.1}$$

where a, b, c are constants.

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Using the traveling wave transformation

$$u=w(z),\quad z=x+ky+lt,$$

into the ZK-MEW equation, it gives a nonlinear ordinary differential equation

$$lw'+a(w^3)'+(blw''+ck^2w'')'=0,$$

and integrating it, we field the following ordinary differential equation

$$(bl+ck^2)w''+aw^3+lw+d=0, \tag{1.2}$$

where a, b, c, d, l, k are constants.

In order to clarify our main results, we need some basic concepts. $w(z)$ is a meromorphic function means that $w(z)$ is holomorphic in the complex plane $\mathbf{C}$ except for poles. We define a meromorphic function f belongs to the class W if f is an elliptic function, or a rational function of $e^{\alpha z}(\alpha\in\mathbf{C})$, or a rational function of z.

Our main results is the following Theorem 1.1.

Theorem 1.1. *Suppose that $a(bl+ck^2)\neq 0$, then all the general meromorphic solutions w of Eq.* (1.2) *are of the following forms:*
(I) The rational function solutions

$$w_{r1}(z)=\pm\sqrt{-\frac{2(bl+ck^2)}{a}}\frac{1}{z-z_0}, \tag{1.3}$$

and

$$w_{r2}(z)=\pm\sqrt{-\frac{2(bl+ck^2)}{az_1^2}}\Big(\frac{z_1}{z-z_0}-\frac{z_1}{z-z_0-z_1}-1\Big), \tag{1.4}$$

where $l=d=0$ in (1.3)*, or*

$$l(z_1^2-6b)=6ck^2,\quad d=\mp 2a\Big(\frac{-2(bl+ck^2)}{az_1^2}\Big)^{\frac{3}{2}},$$

in (1.4)*, $z_0,z_1\neq 0$ are arbitrary complex numbers.*
(II) The simply periodic solutions

$$w_{s1}(z)=\pm\alpha\sqrt{-\frac{bl+ck^2}{2a}}\tanh\frac{\alpha}{2}(z-z_0), \tag{1.5a}$$

$$w_{s2}(z)=\pm\alpha\sqrt{-\frac{bl+ck^2}{2a}}\coth\frac{\alpha}{2}(z-z_0), \tag{1.5b}$$