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General Interpolation Formulae for Barycentric Blending Interpolation

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Abstract. General interpolation formulae for barycentric interpolation and barycentric rational Hermite interpolation are established by introducing multiple parameters, which include many kinds of barycentric interpolation and barycentric rational Hermite interpolation. We discussed the interpolation theorem, dual interpolation and special cases. Numerical example is given to show the effectiveness of the method.

Key Words: General interpolation formulae of interpolation, barycentric interpolation, barycentric rational Hermite interpolation.

AMS Subject Classifications: 41A20, 65D05

1 Introduction

Developing numerical methods for computing approximations of analytic functions by means of polynomials and rational functions represents a fundamental research area of computational mathematics. Lagrangian interpolation, Newton interpolation and Thiele-type continued fractions interpolation may be the favoured linear interpolation and non-linear interpolation. Lagrangian interpolation is praised for analytic utility and beauty but deplored for numerical practice [1]. The advantages of barycentric interpolation formulations in computation are small number of floating point operations (flops) and good numerical stability. Adding a new data pair, the barycentric interpolation formula don't require renew computation all of basis functions [1, 2]. It can avoid the oscillation of Lagrange interpolation by using barycentric interpolation formulations and second kind of Chebyshev points as interpolating points. In barycentric interpolation formulations, the different weight corresponds to different type of interpolation. The most of these interpolation are barycentric rational interpolation. The barycentric rational interpolations have more advantages than the polynomial interpolation and continued fractions interpolation in computation, for example, easy calculation, the information concerning the existence and location of poles of the interpolation, detection of the unattainable

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points, good numerical stability, the usage of shape control [3]. In the last years several researchers have focused their attention on this subject. For example Berrut and Henrici studied barycentric formulas for trigonometric polynomials, barycentric rational formulas [1, 2, 4–6]. Kahng showed the generalizations of univariate Newton's method and applied it to the approximation problems in 1967 [7]. These generalization extended the applicable interpolation functions from polynomials to rational functions, their transformations and some nonlinear functions. Also, these generalizations enabled us to treat the interpolation in a unified manner. Furthermore, Kahng described a class of interpolation functions and showed the explicit method of osculatory interpolation with a function in the class in 1969 [8]. These two functions have many special cases, such as Newton interpolation polynomial, Thiele-type continued fractions interpolation, Hermite interpolation, Salzer-type osculatory interpolation, trigonometric functions interpolations and so on. In 1999, by introducing multiple parameters, Tan and Fang [9] studied several general interpolation formulae for bivariate interpolation which include many classical interpolant schemes, such as bivariate Newton interpolation, Thiele-type branched continued fractions for two variables, Newton-Thiele's blending rational interpolation, Thiele-Newton's blending rational interpolation, and symmetric branched continued fraction discussed by Cuyt and Murphy et al. Tan discussed more general interpolation grids [10]. Recently Tang and Zou [11] have improved and extended the general interpolation formulae studied by Tan and Fang by introducing multiple parameters, so that the new frames can be used to deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur. The general form of block-based bivariate blending rational interpolation with the error estimation is established by introducing two parameters. From the general form, four different block-based interpolations can be obtained. Then an efficient algorithm for computing bivariate lacunary rational interpolation is constructed based on block-based bivariate blending rational interpolation. Tang and Zou [12, 13] construct general structures of one and two variable interpolation function, without depending on the existence of divided differences or inverse differences, and also discusses the block based osculatory interpolation in one variable case, generalize the conclusion of Kahng to bivariate case.

Our contribution in this paper is to obtain general interpolation formulae for barycentric interpolation by introducing multiple parameters, which include Thiele barycentric blending rational interpolation, Newton barycentric blending rational interpolation, associated continued fractions barycentric blending rational interpolation and their dual schemes, bivariate barycentric interpolation, barycentric Thiele blending rational interpolation, barycentric Newton blending rational interpolation, barycentric associated continued fractions blending rational interpolation, barycentric Hermite blending rational interpolation, barycentric Hermite blending rational interpolation based on Padé approximations and so on as its special cases.

The organization of the paper is as follows. In Section 2 we discuss the general interpolation formulae for barycentric blending interpolation and the dual general interpolation formulae and its special cases. In Section 3, we present general interpolation