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Readjustment of the Paper [J. Kaur and S. S. Bhatia, Integrability and L^1-Convergence of Double Cosine Trigonometric Series, Anal. Theory Appl., 27(1) (2011), pp. 32–39.]

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Abstract. In this paper, we show that new modified double cosine trigonometric sums introduced in [1] are inappropriate, the class of double sequences J_d introduced there is unusable for such sums and consequently the results obtained in it are completely incorrect. We here introduce appropriate modified double cosine trigonometric sums making the class J_d usable considering a particular double cosine trigonometric series.

Key Words: L^1-convergence, double null sequence, cosine trigonometric series, modified sums.

AMS Subject Classifications: 42A20, 42B05

1 Introduction and auxiliary statements

For a function $f(x,y)$ with two independent variables x and y we write $f\in L^1(T^2)$ if

$$\|f\|=\iint_{T^2}|f(x,y)|dxdy<+\infty,$$

where $T^2:=[0,\pi]\times[0,\pi]$.

Let

$$f(x,y)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\lambda_j\lambda_k a_{j,k}\cos jx\cos ky \tag{1.1}$$

be a double cosine series on the positive quadrant $T^2:=[0,\pi]\times[0,\pi]$ of the two dimensional torus, where $\lambda_0=1/2$ and $\lambda_i=1$ for $i=1,2,\cdots$, and $\{a_{j,k}\}$ is a double sequence of real numbers.

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Let us denote by

$$S_{m,n}(x,y):=\sum_{j=0}^{m}\sum_{k=0}^{n}\lambda_j\lambda_k a_{j,k}\cos jx\cos ky, \quad m,n\geq 0,$$

the partial sums of the series (1.1) and

$$f(x,y)=\lim_{m+n\to\infty}S_{m,n}(x,y).$$

In 2011, J. Kaur and S. S. Bhatia [1] introduced some new modified double cosine trigonometric sums as follows

$$g_{m,n}(x,y)=\frac{a_{0,0}}{2}+\sum_{j=1}^{m}\sum_{k=1}^{n}\Big(\sum_{i=j}^{m}\sum_{\ell=k}^{n}\Delta_{11}\left(a_{i,\ell}\cos ix\cos \ell y\right)\Big).$$

Also, they defined the following class of numerical sequences.

Definition 1.1. A double null sequence $\{a_{j,k}\}$ of positive numbers is said to belong to the class J_d if there exists a double sequence $\{A_{j,k}\}$ such that

$$A_{j,k}\downarrow 0, \quad j+k\to\infty, \tag{1.2a}$$

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}jkA_{j,k}<\infty, \tag{1.2b}$$

and

$$\left|\Delta_{p,q}\Big(\frac{a_{j,k}}{jk}\Big)\right|\leq\frac{A_{j,k}}{jk}, \quad 1\leq p+q\leq 2, \tag{1.3}$$

for any nonnegative integers p,q and $j,k\in\{1,2,3,\cdots\}$.

Moreover, they have presented the following results:

Theorem 1.1 (see [1]). *If a double sequence $\{a_{j,k}\}$ belongs to the class J_d, then $\|g_{m,n}-f\|\to 0$ as $j+k\to\infty$.*

Corollary 1.1 (see [1]). Under condition of Theorem 1.1, the sum-function f of the series (1.1) is an integrable function and (1.1) is the Fourier series of f.

Corollary 1.2 (see [1]). If a double sequence $\{a_{j,k}\}$ belongs to the class J_d, then $\|S_{m,n}-f\|\to 0$ as $j+k\to\infty$.

Unfortunately, the sums $g_{m,n}(x)$ are not appropriate so that Theorem 1.1, Corollary 1.1 and Corollary 1.2 will be true. Indeed, after some elementary calculations the authors