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On Some Class of n-Normed Generalized Difference Sequences Related to ℓ_p-Space

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Abstract. In this paper, we introduce the class of n-normed generalized difference sequences related to ℓ_p-space. Some properties of this sequence space like solidness, symmetricity, convergence-free etc. are studied. We obtain some inclusion relations involving this sequence space.

Key Words: Generalized difference operator, n-norm, n-Banach space, symmetricity, solidness, convergence free, completeness.

AMS Subject Classifications: 40A05, 40A25, 40A30, 40C05

1 Introduction

The notion of n-normed space was studied at the initial stage by Gahler [7], Misiak [10], Gunawan [8] and many others from different aspects.

Let $n \in N$ and X be a real vector space. A real valued function on X^n satisfying the following $\|\cdot,\cdots\|$ four properties:

1. $\|(z_1,z_2,\cdots,z_n)\|_n = 0$ if and only if $z_1,z_2,\cdots,z_n$ are linearly dependent;
2. $\|(z_1,z_2,\cdots,z_n)\|_n$ is invariant under permutation;
3. $\|(z_1,z_2,\cdots,z_{n-1},\alpha z_n)\|_n = \|\alpha\|\|(z_1,z_2,\cdots,z_n)\|_n$, for all $\alpha \in R$;
4. $\|(z_1,z_2,\cdots,z_{n-1},x+y)\|_n \le \|(z_1,z_2,\cdots,z_{n-1},x)\|_n + \|(z_1,z_2,\cdots,z_{n-1},y)\|_n$;

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is called an n-norm on X and the pair $(X,\|,\cdot,\cdot\|_n)$ is called an n-normed space.

Kizmaz [9] studied the notion of difference sequence spaces at the initial stage. Kizmaz [9] investigated the difference sequence spaces $\ell_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$ of crisp sets. The notion is defined as follows:

$$Z(\Delta)=\{x=(x_k):(\Delta x_k)\in Z\},$$

for $Z=\ell_\infty, c$ and c_0, where $\Delta x=(\Delta x_k)=(x_k-x_{k+1})$, for all $k\in N$. The above spaces are Banach spaces, normed by

$$\|x\|_\Delta=\|x_1\|+\sup_k\|\Delta x_k\|.$$

The idea of Kizmaz [9] was applied to introduce different types of difference sequence spaces and study their different properties by Tripathy (see [13, 14]), Tripathy, Altin and Et [15], Tripathy and Baruah (see [16, 17]), Tripathy abd Borgohain [18], Tripathy and Chandra [19], Tripathy, Choudhary and Sarma [20], Tripathy and Dutta [21], Tripathy and Esi (see [22, 23]), Tripathy, Esi and Tripathy [24], Tripathy and Mahanta [25] and many others.

Tripathy and Esi [22] introduced the new type of difference sequence spaces, for fixed $m\in N$,

$$Z(\Delta_m)=\{x=(x_k):(\Delta_m x_k)\in Z\},$$

for $Z=\ell_\infty, c$ and c_0, where $\Delta_m x=(\Delta_m x_k)=(x_k-x_{k+m})$, for all $k\in N$.

This generalizes the notion of difference sequence spaces studied by Kizmaz [9]. The above spaces are Banach spaces, normed by

$$\|x\|_{\Delta_m}=\sum_{r=1}^{m}\|x_r\|+\sup_k\|\Delta_m x_k\|.$$

Tripathy, Esi and Tripathy [24] further generalized this notion and introduced the following notion. For $m\geq 1$ and $n\geq 1$,

$$Z(\Delta_m^n)=\{x=(x_k):(\Delta_m^n x_k)\in Z\},$$

for $Z=\ell_\infty, c$ and c_0.

This generalized difference has the following binomial representation,

$$\Delta_m^n x_k=\sum_{r=0}^{n}(-1)^r\binom{n}{r}x_{k+rm}. \tag{1.1}$$

Sargent [12] introduced the crisp set sequence space $m(\phi)$ and studied some properties of this space. Later on it was studied from the sequence space point of view and some matrix classes were characterized with one member as $m(\phi)$ by Rath and Tripathy [11]. Afterwards the notion was further investigated by Esi [5], Tripathy and Borgohain [18], Tripathy and Sen [27] and others. In this article we introduce the class of sequences $(m(\phi,\Delta_p^q),\|\cdots,\cdot\|_n)$ with respect to n-norm.