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# Certain Integral Transforms of Generalized $k$-Bessel Function

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**Abstract.** The objective of this note is to provide some (potentially useful) integral transforms (for example, Euler, Laplace, Whittaker etc.) associated with the generalized $k$-Bessel function defined by Saiful and Nisar [3]. We have also discussed some other transforms as special cases of our main results.

**Key Words**: Gamma function, $k$-Bessel function, generalized $k$-Bessel function, integral transforms.

**AMS Subject Classifications**: 33C10, 65R10, 26A33

## 1 Introduction

The Bessel function of fist kind has the power series representation of the form [4]:

$$J_v(z)=\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{2}\right)^{2k+v}}{\Gamma(k+v+1)k!}, \tag{1.1}$$

Romero et al. [16] introduced the $k$-Bessel function of the first kind defined by the series

$$J_{k,v}^{\gamma,\lambda}(x)=\sum_{n=0}^{\infty}\frac{(\gamma)_{n,k}}{\Gamma_k(\lambda n+v+1)}\frac{(-1)^n(x/2)^n}{(n!)^2}, \tag{1.2}$$

where $k\in\mathbb{R}$; $\alpha,\lambda,\gamma,v\in C$; $\Re(\lambda)>0$ and $\Re(v)>0$.

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Very recently, Saiful and Nisar [3] gave a new generalization of $k$-Bessel function called the generalized $k$-Bessel function of the first kind defined for $k\in\mathbb{R}$; $\sigma,\gamma,v,c,b\in\mathbb{C}$; $\Re(\sigma)>0$, $\Re(v)>0$ as:

$$J_{k,v}^{b,c,\gamma,\sigma}(z)=\sum_{n=0}^{\infty}\frac{(c)^n(\gamma)_{n,k}}{\Gamma_k\left(\sigma n+v+\frac{b+1}{2}\right)}\frac{(z/2)^{v+2n}}{(n!)^2},\tag{1.3}$$

where the $k$-Pochhammer symbol $(\gamma)_{n,k}$ is defined by [1]:

$$(\gamma)_{\nu,k}=\frac{\Gamma_k(\gamma+\nu k)}{\Gamma_k(\gamma)},\quad (\gamma\in\mathbb{C}\setminus\{0\}),\tag{1.4}$$

and the $k$-gamma function has the relation

$$\Gamma_k(z)=k^{\frac{z}{k}-1}\Gamma\left(\frac{z}{k}\right),\tag{1.5}$$

such that $\Gamma_k(z)\to\Gamma(z)$ if $k\to 1$.

The generalized hypergeometric function represented as follows [6]:

$${}_pF_q\left[\begin{array}{c}(\alpha_p),\\ (\beta_q),\end{array} z\right]=\sum_{n=0}^{\infty}\frac{\Pi_{j=1}^{p}(\alpha_j)_n}{\Pi_{j=1}^{q}(\beta_j)_n}\frac{z^n}{n!},\tag{1.6}$$

provided $p\le q$, $p=q+1$ and $|z|<1$ and $(\alpha)_n$ is well known Pochhammer symbol (see [6]). The Fox-Wright generalization ${}_p\Psi_q(z)$ of hypergeometric function ${}_pF_q$ is given by (c.f. [7–9,15]):

$$\begin{aligned}{}_p\Psi_q\left[\begin{array}{c}(\alpha_1,A_1),\cdots,(\alpha_p,A_p),\\ (\beta_1,B_1),\cdots,(\beta_q,B_q),\end{array} z\right]&={}_p\Psi_q\left((\alpha_j,A_j)_{1,p};(b_j,\beta_j)_{1,q};z\right)\\ &=\sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+A_1n)\cdots\Gamma(\alpha_p+A_pn)}{\Gamma(\beta_1+B_1n)\cdots\Gamma(\beta_q+B_qn)}\frac{z^n}{n!},\end{aligned}\tag{1.7}$$

where $A_j>0$ $(j=1,2,\cdots,p)$; $B_j>0$ $(j=1,2,\cdots,q)$ and

$$1+\sum_{j=1}^{q}B_j-\sum_{j=1}^{p}A_j\ge 0$$

for suitably bounded value of $|z|$.

The generalized $k$-Wright function introduced in [10] as: For $k\in\mathbb{R}^+$; $z\in\mathbb{C}$, $\alpha_i,\beta_j\in\mathbb{R}(\alpha_i,\beta_j\neq 0;\, i=1,2,\cdots,\, p;j=1,2,\cdots,q)$ and $(a_i+\alpha_i n),(b_j+\beta_j n)\in\mathbb{C}\backslash k\mathbb{Z}^-$

$${}_p\Psi_q^k(z)={}_p\Psi_q^k\left[\left(\begin{array}{cc}(a_i,\alpha_i)_{1,p} & \\ (b_j,\beta_j)_{1,q} & |z\end{array}\right)\right]=\sum_{n=0}^{\infty}\frac{\prod\limits_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod\limits_{j=}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}.\tag{1.8}$$

Also, we recall here the following definitions: