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Certain Results for the 2-Variable Peters Mixed Type and Related Polynomials

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Abstract. In this article, the 2-variable general polynomials are taken as base with Peters polynomials to introduce a family of 2-variable Peters mixed type polynomials. These polynomials are framed within the context of monomiality principle and their properties are established. Certain summation formulae for these polynomials are also derived. Examples of some members belonging to this family are considered and numbers related to some mixed special polynomials are also explored.

Key Words: 2-Variable general polynomials, Peters polynomials, 2-variable truncated exponential polynomials, Sheffer sequences, monomiality principle.

AMS Subject Classifications: 33C45, 33C50, 33E20

1 Introduction and preliminaries

Special functions (or special polynomials) because of their remarkable properties known as "useful functions" and have been used for centuries. In the past years, the development of new special functions and of applications of special functions to new areas of mathematics have initiated a revival of interest in the p-adic analysis, q-analysis, analytic number theory, combinatorics and so on. Moreover, in recent years, the generalized and multi-variable forms of the special functions of mathematical physics have witnessed a significant evolution. In particular, the special polynomials of two variables provided new means of analysis for the solution of large classes of partial differential equations often encountered in physical problems see for example [3,4,6,7,11]. Most of the special functions of mathematical physics and their generalizations have been suggested by physical problems. Also these polynomials are helpful in introducing new families of special polynomials.

Motivated by the importance of the special functions of two variables in applications, a general class of the 2-variable polynomials, namely the 2-variable general polynomials

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Table 1: List of special cases of 2VGP $p_n(x,y)$.

S. No.	$\phi(y,t)$	Generating Functions	Polynomials
I.	e^{yt^m}	$\exp(xt+yt^m)=\sum\limits_{n=0}^{\infty} H_n^{(m)}(x,y)\frac{t^n}{n!}$	Gould-Hopper polynomials [10]
II.	e^{yt^2}	$\exp(xt+yt^2)=\sum\limits_{n=0}^{\infty} H_n(x,y)\frac{t^n}{n!}$	2-variable Hermite Kampé de Feriet polynomials [2]
III.	$C_0(-yt^m)$	$e^{xt}C_0(-yt^m)=\sum\limits_{n=0}^{\infty} {}_mL_n(y,x)\frac{t^n}{n!}$	2-variable generalized Laguerre polynomials [7]
IV.	$C_0(-yt)$	$e^{xt}C_0(-yt)=\sum\limits_{n=0}^{\infty} L_n(y,x)\frac{t^n}{n!}$	2-variable Laguerre polynomials [4]
V.	$\frac{1}{(1-yt^r)}$	$\frac{e^{xt}}{(1-yt^r)}=\sum_{n=0}^{\infty} e_n^{(r)}(x,y)t^n$	2-variable truncated exponential polynomials [8] (of order r)
VI.	$\frac{1}{(1-yt^2)}$	$\frac{e^{xt}}{(1-yt^2)}=\sum_{n=0}^{\infty} {}_{[2]}e_n(x,y)t^n$	2-variable truncated exponential polynomials [6]

(2VGP) $p_n(x,y)$ is considered in [11]. These polynomials are defined by the generating function [11, p.4 (14)]

$$e^{xt}\phi(y,t)=\sum_{n=0}^{\infty} p_n(x,y)\frac{t^n}{n!}, \quad p_0(x,y)=1, \tag{1.1}$$

where $\phi(y,t)$ has (at least the formal) series expansion

$$\phi(y,t)=\sum_{n=0}^{\infty}\phi_n(y)\frac{t^n}{n!}, \quad \phi_0(y)\neq 0. \tag{1.2}$$

The 2VGP family contains very important polynomials such as the Gould-Hopper polynomials (GHP) $H_n^{(m)}(x,y)$, 2-variable Hermite Kampé de Feriet polynomials (2VHKdFP) $H_n(x,y)$, 2-variable generalized Laguerre polynomials (2VGLP) ${}_mL_n(y,x)$, 2-variable Laguerre polynomials (2VLP) $L_n(y,x)$, 2-variable truncated exponential polynomials (of order r) (2VTEP) $e_n^{(r)}(x,y)$ and 2-variable truncated exponential polynomials (2VTEP) ${}_{[2]}e_n(x,y)$. We present the list of some known 2VGP family in Table 1.

It is shown in [11], that the polynomials $p_n(x,y)$ are quasi-monomial [4, 19] with respect to the following multiplicative and derivative operators:

$$\hat{M}_p=x+\frac{\phi'(y,\partial_x)}{\phi(y,\partial_x)}\Big(\partial_x:=\frac{\partial}{\partial x} \quad \text{and} \quad \phi'(y,t):=\frac{\partial}{\partial t}\phi(y,t)\Big) \tag{1.3}$$

and

$$\hat{P}_p=\partial_x, \tag{1.4}$$

respectively.

According to the monomiality principle and in view of Eqs. (1.3) and (1.4), we have

$$\hat{M}_p\{p_n(x,y)\}=p_{n+1}(x,y) \tag{1.5}$$