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Fixed Point Curve for Weakly Inward Contractions and Approximate Fixed Point Property

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Abstract. In this paper, we discuss the concept of fixed point curve for linear interpolations of weakly inward contractions and establish necessary condition for a nonexpansive mapping to have approximate fixed point property.

Key Words: Nonexpansive mapping, contraction, weakly inward contraction, approximate fixed point property.

AMS Subject Classifications: 46B20, 47H07, 47H10, 54E40, 54H99

1 Introduction

The most well known result in the theory of fixed point is Banach contraction principle. It has been used to develop much of the rest of fixed point theory. Another key result in the field is a theorem due to Browder, Ghde, and Kirk involving Hilbert spaces and nonexpansive mappings. In 1988, Sam B. Nadler and K. Ushijima introduced the concept of fixed point curves for linear interpolations of contraction mappings using Banach contraction principle. Here we shall establish the concept of fixed point curves for linear interpolations of weakly inward contractions using the result [2,4], which is an extension of the Banach contraction principle for non-self contraction mappings.

Before introduce fixed point curve and approximate fixed point property for weakly inward contractions, we will give some preliminary definitions and theorems.

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2 Preliminaries

Definition 2.1 (see [1]). Let K be a closed convex subset of a Banach space X and let $C\subset K$. Then a mapping $T:C\to X$ is said to be *inward mapping (respectively, weakly inward)* on C relative to K if $Tx\in I_K(x)$ (repectively, $Tx\in\overline{I_K(x)}$) for $x\in C$, where

$$I_K(x)=\big\{(1-t)x+ty\colon\, y\in K \ \text{ and } \ t\geq 0\big\},$$

and $\overline{I_K(x)}$ is a closure of the $I_K(x)$ in X.

Lemma 2.1 (see [1]). *$\overline{I_K(x)}$ is a closed convex set containing K for each $x\in K$.*

Definition 2.2 (see [1]). Let K be a closed convex set. A map $T\colon\, C\subseteq K\to X$ is said to be *generalized inward* on C relative to K if the following condition is satisfied

$$d(Tx,K)<\|x-Tx\|,\quad \forall x\in C\quad \text{with } \ Tx\notin K,$$

where $d(y,K)=\inf\{\|y-u\|\colon\, u\in K\}$.

Theorem 2.1 (see [3]). *Let C be a non empty closed convex subset of a Banach space X and $T\colon\, C\to X$ a weakly inward contraction mapping. Then T has a unique fixed point in C.*

Proposition 2.1 (see [3]). Let C be a non empty closed convex subset of a Banach space X and $T\colon\, C\to X$ a non expansive mapping that is weakly inward. Then for $u\in C$ and $t\in(0,1)$ there exist one point $x_t\in C$ such that $x_t=(1-t)u+tTx_t$. If C is bounded then $x_t-Tx_t\to 0$ as $t\to 1$.

Corollary 2.1 (see [3]). Let C be a non-empty closed convex (or star shaped) and bounded subset of a Banach space X and $T\colon\, C\to C$ a non expansive mapping. Then there exist a sequence $\{x_n\}$ in C, such that

$$\lim_{n\to\infty}\|x_n-Tx_n\|=0.$$

Definition 2.3 (see [2]). Let (X,d) be a metric space and $f\colon\, X\to X$. Then f has the *approximate fixed point property (a.f.p.p)* if $\forall\epsilon>0$, $F_\epsilon(f)\neq\phi$.

Definition 2.4 (see [2]). Let (X,d) be a metric space. A mapping $f\colon\, X\to X$ is said to be asymptotically regular if $d(f^n(x),f^{n+1}(x))\to 0$ as $n\to\infty$ and $\forall x\in X$.

Lemma 2.2 (see [2]). *Let (X,d) be a metric space and $f\colon\, X\to X$ be such that f is asymptotically regular. Then f has approximate fixed point property.*