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Stability Results for Jungck-Kirk-Mann and Jungck-Kirk Hybrid Iterative Algorithms

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Abstract. In this paper, we introduce two hybrid iterative algorithms of Jungck-Kirk-Mann (J-K-M) and Jungck-Kirk (J-K) types to obtain some stability results for non-selfmappings by employing a certain general contractive condition. Our results generalize and extend most of the existing ones in the literature.

Key Words: Stability result, Jungck-Kirk-Mann, Jungck-Kirk.

AMS Subject Classifications: 47H06, 54H25

1 Introduction

Let (E,d) be a complete metric space, and for $T:E\to E$ a selfmap of E, let

$$F_T=\{p\in E|Tp=p\}$$

be the set of fixed points of T. Also, for $S,T:Y\to E$, let $C(S,T)=\{z\in Y|Sz=Tz=p\}$ be the set of all coincidence points of S and T.

Definition 1.1. (see [20]) Let $S,T:Y\to E, T(Y)\subseteq S(Y)$ and z a coincidence point of S and T, that is,

$$Sz=Tz=p.$$

For any $x_0\in Y$, let the sequence $\{Sx_n\}_{n=0}^{\infty}$ generated by the iterative procedure

$$Sx_{n+1}=f(T,x_n),\quad n=0,1,\cdots, \tag{1.1}$$

converge to p. Let $\{Sy_n\}_{n=0}^{\infty}\subset E$ be an arbitrary sequence, and set

$$\epsilon_n=d\big(Sy_{n+1},f(T,y_n)\big),\quad n=0,1,\cdots.$$

Then, the iterative procedure (1.1) will be called (S,T)-stable if and only if $\lim_{n\to\infty}\epsilon_n=0$ implies that $\lim_{n\to\infty}Sy_n=p$.

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This definition reduces to that of the stability of iterative process in the sense of Harder and Hicks [6] when $Y=E$ and $S=I$ (identity operator).

Remark 1.1. (i) If in (1.1), for $x_0\in E$,

$$Sx_{n+1}=f(T,x_n)=(1-\alpha_n)Sx_n+\alpha_n Tx_n,\quad n=0,1,\cdots,\quad \alpha_n\in[0,1], \tag{1.2}$$

then we get the iterative process of Singh et al. [25].

(ii) If in (1.1), for $x_0\in E$, $Y=E$, $S=I$ (identity operator), then we obtain

$$x_{n+1}=f(T,x_n)=(1-\alpha_n)x_n+\alpha_n Tx_n,\quad n=0,1,\cdots,\quad \alpha_n\in[0,1], \tag{1.3}$$

which is known as the Mann iterative process (see Mann [11]).

(iii) Also, if in (1.1), for $x_0\in E$, $Y=E$, $S=I$ (identity operator), we have

$$x_{n+1}=f(T,x_n)=\sum_{i=0}^{k}\alpha_i T^i x_n,\quad \sum_{i=0}^{k}\alpha_i=1,\quad n=0,1,\cdots, \tag{1.4}$$

where k is a fixed integer and $\alpha_i\geq 0$, $\alpha_0\neq 0$, $\alpha_i\in[0,1]$, and (1.4) is the Kirk's iterative process [9].

For several stability results that have been obtained by various authors and different contractive definitions, we refer to Berinde [3], Harder and Hicks [6], Osilike [14], Rhoades [17,18] and others in the reference of this paper.

We introduce the following hybrid iterative algorithms to establish our results:

Let $(E,\|.\|)$ be a normed linear space, $S,T:Y\to E$ and $T(Y)\subseteq S(Y)$. Then, for $x_0\in Y$, consider the sequence $\{Sx_n\}_{n=0}^{\infty}\subset E$ defined by

$$Sx_{n+1}=\alpha_{n,0}Sx_n+\sum_{i=1}^{k}\alpha_{n,i}T^i x_n,\quad n=0,1,\cdots,\quad \sum_{i=0}^{k}\alpha_{n,i}=1, \tag{1.5}$$

$\alpha_{n,i}\geq 0$, $\alpha_{n,0}\neq 0$, $\alpha_{n,i}\in[0,1]$, where k is a fixed integer.

If in (1.5), $\alpha_{n,i}=\alpha_i$, then we obtain the following interesting iterative scheme: For $x_0\in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}\subset E$ by

$$Sx_{n+1}=\alpha_0 Sx_n+\sum_{i=1}^{k}\alpha_i T^i x_n,\quad n=0,1,\cdots,\quad \sum_{i=0}^{k}\alpha_i=1, \tag{1.6}$$

$\alpha_i\geq 0$, $\alpha_0\neq 0$, $\alpha_i\in[0,1]$, where k is a fixed integer.

Remark 1.2. The scheme defined in (1.5) shall be called the Jungck-Kirk-Mann iterative algorithm while that of (1.6) shall be called the Jungck-Kirk iterative algorithm.