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UNIFORM MEYER SOLUTION TO THE THREE DIMENSIONAL CAUCHY PROBLEM FOR LAPLACE EQUATION

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Abstract. We consider the three dimensional Cauchy problem for the Laplace equation

$$\begin{cases} u_{xx}(x,y,z)+u_{yy}(x,y,z)+u_{zz}(x,y,z)=0, & x\in\mathbf{R}, y\in\mathbf{R}, 0<z\leq 1, \\ u(x,y,0)=g(x,y), & x\in\mathbf{R}, y\in\mathbf{R}, \\ u_z(x,y,0)=0, & x\in\mathbf{R}, y\in\mathbf{R}, \end{cases}$$

where the data is given at $z=0$ and a solution is sought in the region $x,y\in\mathbf{R}, 0<z<1$. The problem is ill-posed, the solution (if it exists) doesn't depend continuously on the initial data. Using Galerkin method and Meyer wavelets, we get the uniform stable wavelet approximate solution. Furthermore, we shall give a recipe for choosing the coarse level resolution.

Key words: *Laplace equation, wavelet solution, uniform convergence*

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1 Introduction

Many physical and engineering problems require the solution of the following Cauchy problem for Laplace equation:

$$\begin{cases} u_{xx}(x,y,z)+u_{yy}(x,y,z)+u_{zz}(x,y,z)=0, & x\in\mathbf{R}, y\in\mathbf{R}, 0<z\leq 1, \\ u(x,y,0)=g(x,y), & x\in\mathbf{R}, y\in\mathbf{R}, \\ u_z(x,y,0)=0, & x\in\mathbf{R}, y\in\mathbf{R}. \end{cases} \tag{1.1}$$

Wavelet regularization methods for solving the Cauchy problem for Laplace Equation have been studied by many authors. They used the wavelet method to approximate the Laplace Equation by Meyer wavelets (see [1]-[2]), but most authors concentrated on the two dimensional case. In this paper, we consider the three dimensional Cauchy problem for Laplace Equation.

To the authors' knowledge, so far there are many papers on the Laplace Equation, but theoretically the error estimates of most regularization methods are in L^2-sense. In this paper, we improve the results and get uniform convergent wavelet solution. We also give a rule for choosing an appropriate wavelet subspace depending on the noise level of the data.

For $v(x,y)\in L^2(\mathbf{R}^2)$, define

$$\| v \|_{L^2}=\left(\int_{\mathbf{R}^2}|v(x,y)|^2\mathrm{d}x\mathrm{d}y\right)^{\frac{1}{2}}, \tag{1.2}$$

and for $v(x,y)\in L^1(\mathbf{R}^2)\bigcap L^2(\mathbf{R}^2)$, define

$$\hat{v}(\xi,\tau)=\frac{1}{2\pi}\int_{\mathbf{R}^2}v(x,y)e^{-i(\xi x+\tau y)}\mathrm{d}x\mathrm{d}y. \tag{1.3}$$

In this paper, $g(x,y)\in L^2(\mathbf{R}^2)$ denotes the accurate data, $g_\delta(x,y)$ denotes the measured data satisfying

$$\| g_\delta(x,y)-g(x,y) \|_{L^2}\leq\delta, \tag{1.4}$$

where δ represents a bound on the measurement error.

Applying Fourier transform with respect to x, y to the problem (1.1), we get

$$\begin{cases} \hat{u}_{zz}(\xi,\tau,z)=(\xi^2+\tau^2)\hat{u}(\xi,\tau,z), & \xi\in\mathbf{R}, \tau\in\mathbf{R}, 0<z\leq 1, \\ \hat{u}(\xi,\tau,0)=\hat{g}(\xi,\tau), & \xi\in\mathbf{R}, \tau\in\mathbf{R}, \\ \hat{u}_z(\xi,\tau,0)=0, & \xi\in\mathbf{R}, \tau\in\mathbf{R}, \end{cases} \tag{1.5}$$

The solution of the problem (1.5) can be expressed by

$$\hat{u}(\xi,\tau,z)=\hat{g}(\xi,\tau)\cosh(\sqrt{\xi^2+\tau^2}z), \tag{1.6}$$