Anal. Theory Appl.
Vol. 27, No. 1 (2011), 21–27
DOI10.1007/s10496-011-0021-y

ON SOME GENERALIZED DIFFERENCE PARANORMED SEQUENCE SPACES ASSOCIATED WITH MULTIPLIER SEQUENCE DEFINED BY MODULUS FUNCTION

Binod Chandra Tripathy

(*Institute of Advanced Study in Science and Technology, India*)

Prabhat Chandra

(*Patna University, India)*

Received May 4, 2009

Abstract. In this article we introduce the paranormed sequence spaces (f,Λ,Δ_m,p), $c_0(f,\Lambda,\Delta_m,p)$ and $\ell_\infty(f,\Lambda,\Delta_m,p)$, associated with the multiplier sequence $\Lambda=(\lambda_k)$, defined by a modulus function f. We study their different properties like solidness, symmetricity, completeness etc. and prove some inclusion results.

Key words: *paranorm, solid space, symmetric space, difference sequence, modulus function, multiplier seuence*

AMS (2010) subject classification: 40A05, 46A45

1 Introduction

Throughout the article w, c, c_0, ℓ_∞ denote the spaces of *all*, *convergent*, *null* and *bounded* sequences, respectively. The zero sequence is denoted by $\theta=(0,\ 0,\ 0,\cdots)$. The scope for the studies on sequence spaces was extended on introducing the notion of an associated multiplier sequence. S. Goes and G. Goes in [3] defined the differentiated sequence space dE and the integrated sequence space $\int E$ for a given sequence space E, by using the multiplier sequence (k^{-1}) and (k), respectively. P.K. Kamthan in [4] used $(k!)$ as the multiplier sequence for studying some sequence spaces. We shall use a general multiplier sequence $\Lambda=(\lambda_k)$ for our study.

The notion of difference sequence was introduced by H. Kizmaz in [5] as follows:

$$Z(\Delta) = \{(x_k) \in w : (\Delta x_k) \in \mathbf{Z}\},$$

for $Z = c, c_0$ and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$, for all $k \in N$.

It was further generalized in [12] as follows:

$$Z(\Delta_m) = \{(x_k) \in w : (\Delta_m x_k) \in \mathbf{Z}\},$$

for $Z = c, c_0$ and ℓ_∞, where $\Delta_m x_k = x_k - x_{k+m}$, for all $k \in \mathbf{N}$.

Throughout the article $p = (p_k)$ is a sequence of strictly positive real numbers. The notion of paranormed sequences was studied by [10] at the initial stage. It was further investigated by [6], [7], [11], [13] and many others.

The notion of modulus function was introduced by Nakano in [8]. It was further investigated with applications to sequence spaces by [1], [9] and many others.

Remark 1.1. It is well known that $\ell_\infty(p) = \ell_\infty$, $c(p) = c$ and $c_0(p) = c_0$ if and only if $0 < h = \inf p_k \le H = \sup p_k < \infty$, (one may refer to [6] and [7]).

2 Definitions and Preliminaries

Definition 2.1. A modulus f is a mapping from $[0,\infty)$ into $[0,\infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$;

(ii) $f(x+y) \le f(x) + f(y)$;

(iii) f is increasing;

(iv) f is continuous from the right at 0.

Hence f is continuous everywhere in $[0,\infty)$.

Definition 2.2. A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$, whenever $(x_k) \in E$ and for all sequences (α_k) of scalars with $|\alpha_k| \le 1$, for all $k \in \mathbf{N}$.

Definition 2.3. A sequence space E is said to be monotone if it contains the canonical preimages of all its step spaces.

Remark 2.1. From the above definitions it is clear that " A sequence space E is solid implies that E is monotone".

Definition 2.4. A sequence space E is said to be symmetric if $(x_{\pi(n)}) \in E$, whenever $(x_n) \in E$, where π is a permutation of N.

Definition 2.5. A sequence space E is said to be convergence free if $(y_n) \in E$, whenever $(x_n) \in E$ and $x_n = 0$ implies $y_n = 0$.