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Analysis of Functionally Graded Piezoelectric Cylinders in a Hygrothermal Environment

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Abstract. This paper presents an analytical solution for the interaction of electric potentials, electric displacement, elastic deformations, and describes hygrothermal effect responses in hollow and solid cylinders, subjected to mechanical load and electric potential. Exact solutions for displacement, stresses and electric potentials in functionally graded piezoelectric material are determined using the infinitesimal theory. The material properties coefficients of the present cylinder are assumed to be graded in the radial direction by a power law distribution. Numerical examples display the significant of influence of material inhomogeneity. It is interesting to note that selecting a specific value of inhomogeneity parameter can optimize the piezoelectric hollow and solid cylinders responses, which will be of particular importance in modern engineering designs.

AMS subject classifications: 74-XX

Key words: Functionally graded, piezoelectric material, hygrothermal effect.

1 Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composite materials, in which the volume fraction of two or more materials is varied smoothly and continuously as a function of position along certain dimension of the structure from one

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point to other [1]. These materials are mainly constructed to operate in high temperature environments.

The functionally graded piezoelectric material (FGPM) is a kind of piezoelectric material with material composition and properties varying continuously in certain direction. The piezoelectric devices can be entirely made of FGPM or use FGPM as a transit inter layer between two different piezoelectric materials. FGPM is composite material intentionally designed to possess desirable properties for some specific applications. The advantage of this new kind of materials can improve the reliability or the service life of piezoelectric devices.

In view of the advantages of FGMs, a number of investigations dealing with thermal stresses had been published in the scientific literature. In recent years, Tanigawa et al. [2] derived a one-dimensional temperature solution for inhomogeneous plate in transient state and also optimized the material composition by introducing a laminated composite model. Zimmerman and Lutz [3] determined the exact solution of thermal stresses and thermal expansions for a uniformly heated FG cylinder. They derived a governing equation for the thermo-elastic equilibrium of the cylinder, by substituting stress strain and kinematic relations into the stress equilibrium equation of cylinder allowing the elastic moduli and thermal expansion coefficient to vary in the radial direction. Then the equation was solved analytically. The mechanical properties of FGMs are often being represented in the exponentially graded form [4–6] and power-law variations one [1, 7–15]. Reddy [1] analyzed the static behavior of FG rectangular plates based on his third-order shear deformation plate theory. Reddy and Chin [10] studied the dynamic thermoelastic response of FG cylinders and plates. In Reddy and Cheng [11], three-dimensional thermomechanical deformations of simply supported, FG rectangular plates were studied by using an asymptotic method.

A new beam element has been developed to study the thermoelastic behavior of FG beam structures by Chakraborty et al. [16] using the first-order shear deformation theory. Nadeau and Ferrari [17] presented a one dimensional thermal stress analysis of a transversely isotropic layer that was inhomogeneous in its thickness. Using the infinitesimal theory of elasticity, Naki and Murat [18] obtained closed-form solutions for stresses and displacements in FG cylindrical and spherical vessels subjected to internal pressure. Dai et al. [19] presented closed form solutions for a FGM hollow cylinder, closed form solutions for the through thickness stresses and perturbation of the magnetic field vector. Allam and Tantawy [20] present an analytical solution for the interaction of electric potentials, electric displacements, elastic deformations and thermoelasticity, and describes electromagnetoelastic responses and perturbation of the magnetic field vector in hollow structures subjected to mechanical load and electric potential. They consider the solution for the case of hollow structure made of viscoelastic isotropic material, reinforced by elastic isotropic fibers; this material is considered as structurally anisotropic material. More reports on FG structures may also be found in the literature, such as [21–23].

The degradation in performance of the structure due to moisture concentration and high temperature has become increasingly more important with the prolonged use of