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A Collocation Method for Solving Fractional Riccati Differential Equation

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Abstract. In this article, we have introduced a Taylor collocation method, which is based on collocation method for solving fractional Riccati differential equation. The fractional derivatives are described in the Caputo sense. This method is based on first taking the truncated Taylor expansions of the solution function in the fractional Riccati differential equation and then substituting their matrix forms into the equation. Using collocation points, the systems of nonlinear algebraic equation is derived. We further solve the system of nonlinear algebraic equation using Maple 13 and thus obtain the coefficients of the generalized Taylor expansion. Illustrative examples are presented to demonstrate the effectiveness of the proposed method.

AMS subject classifications: 34A08, 656N35, 30K05

Key words: Riccati equation, fractional derivative, collocation method, generalized Taylor series, approximate solution.

1 Introduction

The concept of fractional or non-integer order derivation and integration can be traced back to the genesis of integer order calculus itself [1, 2]. Fractional calculus has become the focus of interest for many researchers in different disciplines of science and technology. The fractional differential equations (FDEs) have received considerable interest in recent years. FDEs have shown to be adequate models for various physical phenomena in areas like damping laws, diffusion processes, etc. For example, the nonlinear oscillation of earthquake can be modeled with fractional derivatives [3], the fluid-dynamic traffic model with fractional derivatives [4], psychology [5] and etc. [6–9].

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In this paper, we present numerical and analytical solutions for the fractional Riccati differential equation

$$D^{\alpha}y(x)=A(x)+B(x)y(x)+C(x)y^2(x), \quad x>0, \quad 0<\alpha\le 1, \tag{1.1}$$

subject to the initial conditions

$$y(0)=\lambda, \tag{1.2}$$

where $A(x)$, $B(x)$ and $C(x)$ are given functions, α is a parameter describing the order of the fractional derivative. The general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses. In the case of $\alpha=1$ the fractional equation reduces to the classical Riccati differential equation. The importance of this equation usually arises in the optimal control problems [10]. The existing literature on fractional differential equations tends to focus on particular values for the order α. The value $\alpha=0.5$ is especially popular. This is because in classical fractional calculus, many of the model equations developed used these particular orders of derivatives [11]. In modern applications (see, e.g., [12]) much more general values of the order an appear in the equations and therefore one needs to consider numerical and analytical methods to solve differential equations of arbitrary order. This equation is solve numerically in [13–15]. In [13], it is given numerical solution of approximate solution of linear fractional differential equations with variable coefficients by collocation method. In [14], a modification of He's homotopy perturbation method is presented. In this method, which does not require a small parameter in an equation, a homotopy with an imbedding parameter $p\in[0,1]$ is constructed. In [15], it is implemented a relatively new analytical technique, the Adomian decomposition method. The solution takes the form of a convergent series with easily computable components. The diagonal Pade approximants are effectively used in the analysis to capture the essential behavior of the solution.

We seek by collocation method the approximate solution of Eq. (1.1) under the condition Eq. (1.2) using the fractional Taylor series

$$y_N(x)=\sum_{i=0}^{N}\frac{(x-c)^{i\alpha}}{\Gamma(i\alpha+1)}(D^{i\alpha}y(x))_{x=c}, \tag{1.3}$$

where $0<\alpha\le 1$ and $D^{i\alpha}y(x)\in C(a,b]$. This method transforms each part of equation into a matrix form, and using the collocation points

$$x_i=\frac{i}{N}, \quad i=0,1,\cdots,N, \tag{1.4}$$

we derive the nonlinear algebraic equation. Solving this equation, we obtained the coefficients of the generalized Taylor series and thus the approximate solutions for various N. Recently, collocation method has become a very useful technique for solving equations. For instance, some authors gave the numerical studies for solving linear differential difference equations [16], Volterra integral equations [17], linear integro-differential