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A Diffusively Corrected Multiclass Lighthill-Whitham-Richards Traffic Model with Anticipation Lengths and Reaction Times

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Abstract. Multiclass Lighthill-Whitham-Richards traffic models [Benzoni-Gavage and Colombo, Euro. J. Appl. Math., 14 (2003), pp. 587–612; Wong and Wong, Transp. Res. A, 36 (2002), pp. 827–841] give rise to first-order systems of conservation laws that are hyperbolic under usual conditions, so that their associated Cauchy problems are well-posed. Anticipation lengths and reaction times can be incorporated into these models by adding certain conservative second-order terms to these first-order conservation laws. These terms can be diffusive under certain circumstances, thus, in principle, ensuring the stability of the solutions. The purpose of this paper is to analyze the stability of these diffusively corrected models under varying reaction times and anticipation lengths. It is demonstrated that instabilities may develop for high reaction times and short anticipation lengths, and that these instabilities may have controlled frequencies and amplitudes due to their nonlinear nature.

AMS subject classifications: 35K65, 35L65, 76T99, 76E99

Key words: Traffic flow, multispecies model, anticipation length, reaction time, diffusive correction, quasilinear system of second-order PDEs, stability, numerical simulation.

1 Introduction

1.1 Scope

The well-known Lighthill-Whitham-Richards (LWR) kinematic traffic model [21,31] states that the density of cars $\phi=\rho/\rho_{\max}$, where ρ is the local number of cars per mile and

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$\rho_{\max}$ is some maximum bumper-to-bumper density, can be described by the conservation law $\partial_t\phi+\partial_x(\phi v(\phi))_x=0$, where t is time, x is the spatial coordinate along either an unbounded, one-directional highway or a closed circuit, and the local velocity $v=v(x,t)$ is a given function of the local density, $v=v(\phi(x,t))$. It is usually assumed that $v(\phi)=v^{\max}V(\phi)$, where $v^{\max}$ is the preferential velocity of drivers on a free highway and V is a hindrance function describing the drivers' behaviour of reducing speed in presence of other cars. The function V satisfies $V(0)=1$ and $V'(\phi)\leq 0$. These assumptions lead to the one-dimensional scalar conservation law

$$\partial_t\phi+\partial_x f(\phi)=0, \quad x\in\mathbb{R}, \quad t>0, \tag{1.1}$$

where the flux density function f is given by

$$f(\phi)=\phi v(\phi)=v^{\max}\phi V(\phi). \tag{1.2}$$

The model (1.1), (1.2) has been extended in several directions. On one hand, Nelson [23, 24] showed that introducing an anticipation length L and a reaction time τ, replacing $V(\phi(x,t))$ by $V(\phi(x+L-v^{\max}V\tau,t-\tau))$ and neglecting $\mathcal{O}(L^2+\tau^2)$ terms when expanding the latter expression around (x,t), one obtains a "diffusively corrected" version of (1.1), (1.2) of the following form:

$$\partial_t\phi+\partial_x f(\phi)=A(\phi)_{xx}. \tag{1.3}$$

Here, L may also depend on ϕ, and under certain restrictions on $L=L(\phi)$, τ and $v(\phi)$, the function A is Lipschitz continuous and increasing so that the governing equation (1.3) of the diffusively corrected LWR model ("DCLWR model") is a strongly degenerate parabolic PDE in the sense that $A(\phi)=0$ for $\phi\leq\phi_{\mathrm{c}}$, where ϕ_{c} is a critical density value (e.g., a perception threshold), and $A'(\phi)>0$ for $\phi>\phi_{\mathrm{c}}$. Properties of (1.3), under the additional assumption of abruptly varying road surface conditions, were analyzed in [8]. On the other hand, Benzoni-Gavage and Colombo [3] and Wong and Wong [37] extended the LWR model (1.1), (1.2) to a multi-class model, the so-called "MCLWR model", by distinguishing N classes of drivers associated with preferential velocities $v_1^{\max}>v_2^{\max}>\cdots>v_N^{\max}$. For the MCLWR model, the sought quantity is the vector $\Phi:=(\phi_1,\cdots,\phi_N)^{\mathrm{T}}$ of the densities ϕ_i of the cars of the different driver classes. The local velocity v_i of vehicles of driver class i is given by $v_i=v_i(\phi)=v_i^{\max}V(\phi)$ for $i=1,\cdots,N$, where we define $\phi:=\phi_1+\cdots+\phi_N$. Thus, the MCLWR model is given by a strongly coupled system of nonlinear first-order conservation laws of the type

$$\partial_t\Phi+\partial_x \boldsymbol{f}(\Phi)=\mathbf{0}, \quad x\in\mathbb{R}, \quad t>0; \quad \boldsymbol{f}(\Phi)=\big(f_1(\Phi),\cdots,f_N(\Phi)\big)^{\mathrm{T}}, \tag{1.4}$$

where the components of the flux vector $\boldsymbol{f}(\Phi)$ are given by

$$f_i(\Phi)=\phi_i v_i(\phi)=\phi_i v_i^{\max}V(\phi), \quad i=1,\cdots,N. \tag{1.5}$$