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Analysis of Two-Phase Cavitating Flow with Two-Fluid Model Using Integrated Boltzmann Equations

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Abstract. In the present work, both computational and experimental methods are employed to study the two-phase flow occurring in a model pump sump. The two-fluid model of the two-phase flow has been applied to the simulation of the three-dimensional cavitating flow. The governing equations of the two-phase cavitating flow are derived from the kinetic theory based on the Boltzmann equation. The isotropic RNG $k-\epsilon-k_{ca}$ turbulence model of two-phase flows in the form of cavity number instead of the form of cavity phase volume fraction is developed. The RNG $k-\epsilon-k_{ca}$ turbulence model, that is the RNG $k-\epsilon$ turbulence model for the liquid phase combined with the k_{ca} model for the cavity phase, is employed to close the governing turbulent equations of the two-phase flow. The computation of the cavitating flow through a model pump sump has been carried out with this model in three-dimensional spaces. The calculated results have been compared with the data of the PIV experiment. Good qualitative agreement has been achieved which exhibits the reliability of the numerical simulation model.

AMS subject classifications: 76T10, 76M12

Key words: Cavitating flow, two-fluid model, RNG $k-\epsilon-k_{ca}$ turbulence model, Boltzmann equations, kinetic theory.

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1 Introduction

Cavitating flow is a type of two-phase flow with liquid phase and vapor phase. In some cases, cavitating flow is turbulent, highly dynamic and highly unstable. There is not only momentum transfer between the liquid phase and cavity phase, but also mass transfer, that is, the vaporizing process and the liquidizing process. Because of these mentioned complexties, there are many difficulties in the numerical simulation of cavitating flow compared to the normal silt-liquid and particulate-gas two-phase flow simulation.

In recent years, there has been much progress in cavitating flow simulation. Simulation methods have been developed from inviscid flow calculation to viscous flow calculation, from two-dimensional computation to three-dimensional computation, and from single-phase flow simulation to two-phase flow simulation.

Beginning in the 1960s and 1970s, many cavitating flow models have been established based on the ideal fluid assumption and the singularity method. Yamaguchi and Kato [1] proposed a cavitating flow model, which was used widely in calculation. Brewer and Kinnas [2] used this model to calculate the flow around two-dimensional (2D) and three-dimensional (3D) hydrofoils, and Pellone and Peallat [3] used it to predict the local bubbles near the hydrofoil surface. De Lange and De Bruin [4] numerically simulated the periodic variation of bubbles.

As turbulent flow simulation has developed, it has been extended to the analysis of cavitating flow. Up to now, the most widely used method for this cavitating flow analysis is the single-phase flow model, even though the cavitating flow is actually a two-phase flow consisting of a cavity phase and a liquid phase. This single-phase cavitating flow model numerically models the flow through direct computation of the single-phase Navier-Stokes equations. A possible simplification of this type of complex flow is to assume the gas-liquid flow is a virtual single-phase, with a sharp density change as long as the pressure drops below some critical pressure (Kubota et al. [5]; Song et al. [6]).

The single-phase cavitating flow model is mainly used in fixed bubble flow calculation because the position of a fixed bubble is rather stable from the point view of direct observation. Actually, the time averaged results with this method is rather stable. The bubbles in the flow have variations in their shape, size and length over time. The liquid flow around bubbles is the main flow area, with much greater velocity than that of vapor in bubbles. Thus, in the model, the surfaces of bubbles can be assumed to be solid walls, on which the pressure is equal to the vaporizing pressure at a certain temperature.

In the single-phase simulation, the algorithm first simulates the whole flow field without bubbles; and then judges areas with pressure less than the vaporizing pressure; third, treats these areas as bubble areas; and finally, recalculates the whole flow field again. This procedure is repeated until the iteration is converged.

The single-phase simulation for cavitating flow is simple and easy, because the single turbulent simulation model and numerical method have been well developed nowadays. But its application is limited to fixed-bubble cavitating flow. For other types of cavitating flow, for example, dissociative bubble flow and bubble cloud, it may be difficult to