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Decoupled Scheme for Non-Stationary Viscoelastic Fluid Flow

Md. Abdullah Al Mahbub[1,2], Shahid Hussain[1], Nasrin Jahan Nasu[1] and Haibiao Zheng[1,*]

[1] *School of Mathematical Sciences, East China Normal University, Shanghai Key Laboratory of Pure Mathematics and Mathematical Practice, Shanghai 200241, China*
[2] *Department of Mathematics, Comilla University, Comilla 3506, Bangladesh*

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Abstract. In this paper, we present a decoupled finite element scheme for two-dimensional time-dependent viscoelastic fluid flow obeying an Oldroyd-B constitutive equation. The key idea of our decoupled scheme is to divide the full problem into two subproblems, one is the constitutive equation which is stabilized by using discontinuous Galerkin (DG) approximation, and the other is the Stokes problem, can be computed parallel. The decoupled scheme can reduce the computational cost of the numerical simulation and implementation is easy. We compute the velocity $\mathbf{u}$ and the pressure p from the Stokes like problem, another unknown stress σ from the constitutive equation. The approximation of stress, velocity and pressure are respectively, P_1-discontinuous, P_2-continuous, and P_1-continuous finite elements. The well-posedness of the finite element scheme is presented and derive the stability analysis of the decoupled algorithm. We obtain the desired error bound also demonstrate the order of the convergence, stability and the flow behavior with the support of two numerical experiments which reveals that decoupled scheme is more efficient than coupled scheme.

AMS subject classifications: 65N12, 65N30, 76A10, 76M10

Key words: Viscoelastic fluid, decoupled scheme, DG method, Oldroyd-B fluid flow model.

1 Introduction

In nature, most of the fluids are non-Newtonian which has a great impact on research due to its enormous and significant practical applications. Viscoelastic fluid can be used in various areas such as polymeric industries, biological rheology, daily life uses, petroleum

*Corresponding author.
Emails: dipmahbub13@yahoo.com (M. A. A. Mahbub), shahid8310@yahoo.com (S. Hussain), dipjahan28@yahoo.com (N. J. Nasu), hbzheng@math.ecnu.edu.cn (H. Zheng)

engineering, nuclear industries, blood flow motion in arteries, coating of the polymeric solution, ink-jet printing and so on. The fluid which satisfies the properties of the viscous fluid and the elastic properties of solids are called viscoelastic fluid. The Newtonian fluid has a proportional relationship with the Cauchy stress and strain which appears linearly and the proportional constant known as Newtonian viscosity. On the other hand, the relationship for the non-Newtonian fluid appears in a nonlinear manner. Non-Newtonian fluids are many kinds like inelastic, linear and nonlinear type. Over the last few decades, the developments of the viscoelastic fluid research have been achieved significant progress, but in theoretical, experimental and numerical aspects, the study of the viscoelastic fluid is different from the Newtonian fluid.

Due to the shear rate dependent viscosity, drag representation, stress relaxation and many other complex structures of viscoelastic fluid cause many effects can't be predicted by the Navier-Stokes equation [1, 2]. Over the last century, it was a significant challenge to formulate a suitable constitutive model to describe the large deformation of the viscoelastic fluid and successfully introduced by James G. Oldroyd [3] in 1950 to study the behavior of the dilute solution of a polymeric molecule. Since then many models were developed to study the viscoelastic fluid flow such as Maxwell (UCM) model, Oldroyd-B model, Phan-Thien-Tanner (PTT) model, Larson model, Johnson-Segalman model, and so on.

The difficulty arises to approximate the viscoelastic fluid flow model by the hyperbolic nature of the constitutive equation requires stabilization in computation. Accurate numerical simulation is essential for the transient viscoelastic fluid flow to understand many problems in non-Newtonian fluid mechanics, particularly those related to flow instabilities [3–5]. The underlying equations are usually considered as the (parabolic) conservation of momentum and incompressibility equations for fluid flow, coupled with a (hyperbolic) constitutive equation for the viscoelastic component of the stress. Some existence results for the viscoelastic flow with a differential constitutive law is obtained by Guillopé and Saut in [6], more complete discussion of existence and uniqueness issues can be found in [1].

In 1973, Reed and Hill first introduced discontinuous Galerkin (DG) method to study the hyperbolic equation in [7]. After one year, Lesaint and Raviart gave the analysis of this method for hyperbolic PDEs to solve the neutron transport equation in [8]. DG method became popular due to its computational flexibility, ability to incorporate physical properties, element-wise conservative and implementable on an unstructured mesh. In 1986, Johnson and Pitkäranta analyzed the DG method for a scalar hyperbolic equation in [9]. DG method for the viscoelastic fluid flow was first introduced by Fortin et al. in [10, 11] but their decoupled steady-state scheme didn't converge. Later, Atkins and Shu gave a quadrature-free implementation of DG method for the hyperbolic equation in [12]. To avoid the introduction of spurious oscillations in finite element approximation for the constitutive equation in [13], Baranger and Wardi used the implicit Euler temporal discretization and the discontinuous Galerkin (DG) approximation for the hyperbolic constitutive equation, required certain time step-size conditions $k=\mathcal{O}(h^{3/2})$. Uncondi-