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Finite Element Solution for MHD Flow of Nanofluids with Heat and Mass Transfer through a Porous Media with Thermal Radiation, Viscous Dissipation and Chemical Reaction Effects

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Abstract. In this paper, the problem of magnetohydrodynamics (MHD) boundary layer flow of nanofluid with heat and mass transfer through a porous media in the presence of thermal radiation, viscous dissipation and chemical reaction is studied. Three types of nanofluids, namely Copper (Cu)-water, Alumina (Al_2O_3)-water and Titanium Oxide (TiO_2)-water are considered. The governing set of partial differential equations of the problem is reduced into the coupled nonlinear system of ordinary differential equations (ODEs) by means of similarity transformations. Finite element solution of the resulting system of nonlinear differential equations is obtained using continuous Galerkin-Petrov discretization together with the well-known shooting technique. The obtained results are validated using MATLAB "bvp4c" function and with the existing results in the literature. Numerical results for the dimensionless velocity, temperature and concentration profiles are obtained and the impact of various physical parameters such as the magnetic parameter M, solid volume fraction of nanoparticles ϕ and type of nanofluid on the flow is discussed. The results obtained in this study confirm the idea that the finite element method (FEM) is a powerful mathematical technique which can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering.

AMS subject classifications: 35A25, 65M60, 76D05

Key words: Boundary layer flow, nanofluid, magnetohydrodynamics, stretching sheet, porous medium, heat and mass transfer, finite element method (FEM), continuous Galerkin-Petrov (cGP) discretization, numerical comparison.

1 Introduction

The study of boundary layer flow over a stretching plate received a great attention in the recent years in view of its significant applications in several engineering processes

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such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, polymer extrusion and metal spinning. The study of hydrodynamic flow and heat transfer over a stretching sheet may find its application to sheet extrusion in order to make flat plastic sheets. Sakiadis [1–3] was the first who investigated the boundary layer flow of an incompressible fluid over a stretching sheet with constant speed from a slit. He employed a similarity transformations in order to obtain the numerical solution for the problem. Tsou et al. [4] investigated the effect of heat transfer in the boundary layer flow over a moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis [1]. Crane [5] made the extension in [1] and considered the problem of a stretching sheet where the velocity is proportional to the distance from the slit. He found the closed form exponential solution for the planar viscous flow with linear stretching case. Chakrabarti et al. [6] discussed the hydromagnetic flow and heat transfer over a stretching sheet. Grubka et al. [7] analyzed heat transfer over a continuous, stretching surface with variable temperature. Andreson et al. [8] extended the work of Crane [5] and investigated the magnetohydrodynamic flow of power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field by using an exact similarity transformations. The effect of mixed convection adjacent to vertical, continuously stretching sheet has been discussed by Chen [9]. Vajravelu et al. [10] investigated the hydromagnetic flow of a second grade fluid over a stretching sheet. Cortell [11] analyzed the magnetohydrodynamic flow of a power-law fluid over a stretching sheet. Abel et al. [12] studied the flow and heat transfer in a MHD viscoelastic boundary layer flow over a stretching sheet with both prescribed surface temperature (PST) and prescribed heat flux (PHF) cases with the effects of non-uniform heat source and radiation. Aziz [13] obtained the similarity solution for laminar thermal boundary layer flow over a plate with a convective surface boundary condition using the symbolic algebra software MAPLE.

Heating and cooling of fluids are very important in many industrial applications such as power, manufacturing, transportation and electronics. Therefore, effective cooling techniques are required for the cooling of high-energy devices. Common heat transfer process in standard fluids, for example, water, ethylene glycol and engine oil have limited/poor heat transfer capabilities because of their low heat transfer properties. In contrast, metals have good thermal conductivities up to three times higher than these fluids [14], so it is desired to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. A lot of experimental and theoretical researches have been made in order to improve the thermal conductivity of such fluids. In 1993, during an investigation of new coolants and cooling technologies at Argonne National Laboratory (ANL) in U.S, Choi invented a new type of fluid known as nanofluid [15]. Nanofluids are fluids that contain small volumetric quantities of nanometer-sized particles, called nanoparticles or nanofibers. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, carbon, nitrides, etc. The most common base fluids include water, ethylene glycol and oil. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles in order to enhance the thermal