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A Study of Crack-Face Boundary Conditions for Piezoelectric Strip Cut Along Two Equal Collinear Cracks

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Abstract. A problem of two equal, semi-permeable, collinear cracks, situated normal to the edges of an infinitely long piezoelectric strip is considered. Piezoelectric strip being prescribed out-of-plane shear stress and in-plane electric-displacement. The Fourier series and integral equation methods are adopted to obtain analytical solution of the problem. Closed-form analytic expressions are derived for various fracture parameters viz. crack-sliding displacement, crack opening potential drop, field intensity factors and energy release rate. An numerical case study is considered for poled $PZT-5H, BaTiO_3$ and $PZT-6B$ piezoelectric ceramics to study the effect of applied electro-mechanical loadings, crack-face boundary conditions as well as inter-crack distance on fracture parameters. The obtained results are presented graphically, discussed and concluded.

AMS subject classifications: 65M10, 78A48

Key words: Collinear cracks, Fourier series method, piezoelectric strip, semi-permeable crack.

1 Introduction

Due to intrinsic electro-mechanical coupling effect, piezoelectric materials have vast utility in many engineering devices, such as sensors, transducers, actuator components. But, under the action of electromechanical loadings, these piezoelectric materials could fail prematurely due to defects, e.g., cracks, holes, etc. arising during their manufacturing process over loads/fatigue/aging etc.

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One of the most important and basic issue addressed in fracture behavior of piezoelectric materials is electric crack-face boundary conditions. Three different types of crack-face boundary conditions (permeable, impermeable and semi-permeable) are analyzed and discussed by many researchers. The permeable crack-face boundary condition was proposed by Parton [1], the impermeable crack model was discussed by Deeg [2] and the semi-permeable crack model was developed by Hao and Shen [3].

The semi-permeable electric crack-face boundary condition is very closed to real situation due to the non-zero finite permittivity of crack media, Mathematically, it is expressed as

$$D_n^+(w^+ - w^-) = -\varepsilon_a(\phi^+ - \phi^-), \tag{1.1}$$

where, superscripts $+$ and $-$ indicate the values over the upper and lower crack-faces, respectively. D_n, ϕ and w are the normal component of electric displacement, electric potential and mechanical displacement, respectively.

Hang et al. [4] analyzed the crack-face boundary conditions and found that crack gap dielectric media plays an important role to reduce the singularity of the stress and electric displacement. The analytic closed-form solutions for a semi-permeable crack using Stroh-formalism technique for mode-I as well as for mode-III loadings, respectively, are derived by Zhang and Gao [5]. They also analyzed the fracture and failure behavior of piezoelectric ceramics experimentally and compared both analytic and experiment results. Ou and Chen [6] obtained energy release rate at the crack tips for the semi-permeable crack (filled with different medium, like: air and silicon oil) under very large electromechanical loading conditions and compared obtained results with impermeable crack model. The fundamental solution for a semi-permeable interface crack between two dissimilar piezoelectric materials obtained by Li and Chen [7].

Fracture analysis for a semi-permeable crack model have been addressed by many researchers using various analytical and numerical techniques. The influence of crack-face boundary conditions on stress and electric displacement intensity factor is studied by Zhou et al. [8]. A problem of two collinear unequal semi-permeable cracks in piezoelectric plane under mode-I loading using a new approach of real fundamental solution is addressed by Li and Lee [9]. Their study also included the effects of geometric dimensions, electric loading and electric displacement on energy release rate. More recently, the effect of inter-crack distance as well as crack-face boundary conditions on fracture parameters for collinear semi-permeable mode-I cracks in piezoelectric plane is addressed by Bhargava et al. [10], using Stroh formalism and complex variable technique.

A lot of work has been done on Mode-I multiple cracks problems with semi-permeable boundary conditions, while much less work is reported on Mode-III multiple cracks problems in piezoelectric materials. To address this paucity, we develop a mathematical model for two equal collinear transverse cracks in a piezoelectric strip, under mode-III deformation. And study influence of crack-face boundary conditions on fracture parameters. The analytic closed form expressions are derived for crack-sliding displacement (CSD), crack opening potential drop (COP), field intensity factors (IFs) and energy re-