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A Moving Mesh Method for Kinetic/Hydrodynamic Coupling

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Abstract. This paper deals with the application of a moving mesh method for kinetic/hydrodynamic coupling model in two dimensions. With some criteria, the domain is dynamically decomposed into three parts: kinetic regions where fluids are far from equilibrium, hydrodynamic regions where fluids are near thermodynamical equilibrium and buffer regions which are used as a smooth transition. The Boltzmann-BGK equation is solved in kinetic regions, while Euler equations in hydrodynamic regions and both equations in buffer regions. By a well defined monitor function, our moving mesh method smoothly concentrate the mesh grids to the regions containing rapid variation of the solutions. In each moving mesh step, the solutions are conservatively updated to the new mesh and the cut-off function is rebuilt first to consist with the region decomposition after the mesh motion. In such a framework, the evolution of the hybrid model and the moving mesh procedure can be implemented independently, therefore keep the advantages of both approaches. Numerical examples are presented to demonstrate the efficiency of the method.

AMS subject classifications: 65M50, 76P05

Key words: Moving mesh method, kinetic/hydrodynamic coupling, the Boltzmann-BGK equation.

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1 Introduction

Hydrodynamic equations such as the Navier-Stokes or the Euler equations have achieved successful applications in many areas of fluid dynamics, while there are cases that this hydrodynamic equations do not provide a satisfactory description of the physical system. Then we have to use a kinetic description by the Boltzmann equation or a simplified version of it, i.e., the Boltzmann-BGK equation. However, even nowadays the numerical solution for such microscopic models is too expensive

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to be solved. In most cases, we are primarily concerned about the macroscopic phenomena of the problems rather than the microscopic state which need to be solved through the microscopic model and the hydrodynamic equations can provide a sufficiently accurate description of the problems only except in very local domain where the hydrodynamic equations breakdown, such as shock wave and boundary layers. In this situations, a coupled method of the kinetic/hydrodynamic model can be expected more efficient than the whole microscopic models and still provide the correct representation of the physical phenomena. In the last few years many investigations have been achieved in this direction, such as the coupling of different models and different implementation techniques (e.g., [1–6]). Our purpose of this paper is to study a moving mesh adaptive method for the kinetic/hydrodynamic coupling model.

This paper is a further study of the kinetic/hydrodynamic coupling model introduced in [2] on 2-dimensional domain. The Boltzmann-BGK equation is used as kinetic model on kinetic regions and the Euler equations as hydrodynamic model on hydrodynamic regions respectively. We use Fig. 1 to illustrate the decomposition of the domain. Since the two regions are connected by some fixed-width buffer zones and both of the kinetic and hydrodynamic models are solved on not only their own regions but also the buffer zones, the solutions are combined with both ingredients through a cut-off function. As presented in [1,2], the use of the buffer zones and the cut-off function makes the coupling model can be solved quite smoothly without any interface condition. When the time evolutions, the kinetic regions are automatically generated based on some equilibrium criteria, which indicate the deviation from the current microscopic state to thermodynamical equilibrium if microscopic data are available, or if not, the breakdown of the hydrodynamic model. With this moving zones technique, the kinetic zones can be chosen as small as possible to speedup numerical simulations while preserve the accuracy of the physical phenomena.

Because the solution of the kinetic/hydrodynamic coupling model have large variations over some local domain, it is benefit to use adaptive mesh methods to increase the accuracy and decrease the computational time. In [3], an h-adaptive mesh method is contributed and demonstrate its efficiency comparing to the uniform mesh methods. Encouraged by their result and noticed that a smoothly mesh redistribution strategy

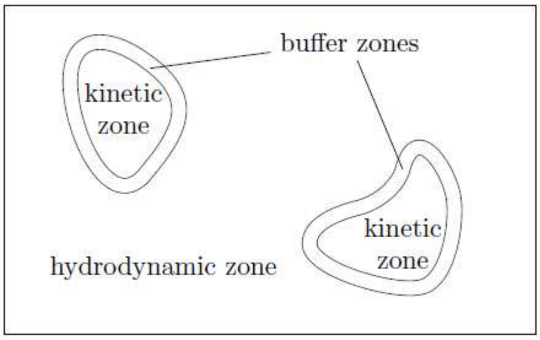

Figure 1: Domain decomposition.