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An Improved Formulation of Singular Boundary Method

Wen Chen*and Yan Gu

College of Harbour, Coastal and Offshore Engineering, Hohai University, No. 1 Xikang Road, Nanjing, Jiangsu 210098, China

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Abstract. This study proposes a new formulation of singular boundary method (SBM) to solve the 2D potential problems, while retaining its original merits being free of integration and mesh, easy-to-program, accurate and mathematically simple without the requirement of a fictitious boundary as in the method of fundamental solutions (MFS). The key idea of the SBM is to introduce the concept of the origin intensity factor to isolate the singularity of fundamental solution so that the source points can be placed directly on the physical boundary. This paper presents a new approach to derive the analytical solution of the origin intensity factor based on the proposed subtracting and adding-back techniques. And the troublesome sample nodes in the ordinary SBM are avoided and the sample solution is also not necessary for the Neumann boundary condition. Three benchmark problems are tested to demonstrate the feasibility and accuracy of the new formulation through detailed comparisons with the boundary element method (BEM), MFS, regularized meshless method (RMM) and boundary distributed source (BDS) method.

AMS subject classifications: 65N15, 65N38

Key words: Singular boundary method, fundamental solution, singularity, desingularization technique, meshless.

1 Introduction

The method of fundamental solution (MFS) [1–3] is one of the collocation based boundary type meshless methods with the merit of easy programming, high accuracy and fast convergence. In order to avoid the singularity of fundamental solutions with a strong-form collocation formulation, the MFS, however, places the source points on a fictitious boundary outside or inside the physical domain, respectively, corresponding to interior or exterior problems. Despite many years of hard research, the placement

*Corresponding author.
URL: `http://em.hhu.edu.cn/chenwen/english.html`
Email: chenwen@hhu.edu.cn (W. Chen), guyan1913@163.com (Y. Gu)

of this fictitious boundary for complex-shaped or multi-connected domain problems remains a tricky art and is determined largely based on a trial-error approach [4–6].

Great efforts have since been made to remove the perplexing issue of fictitious boundary encountered in the MFS. For this purpose, the boundary knot method (BKM) was proposed by Chen et al. [7–10], in which the nonsingular general solution is employed instead of using the singular fundamental solutions. Consequently, this method can place source knots on the physical boundary while being meshless and integration-free. However, similar to the MFS, the condition number of its discretization matrix worsens quickly with an increasing number of boundary nodes. The BKM is also mostly applied to interior Helmholtz and diffusion problems since the nonsingular general solution is not available in some cases such as Laplace equation.

Based on the double-layer potential theory, an alternative collocation strong-form technique, called the regularized meshless method (RMM), was proposed by Young and his coworkers [11, 12]. This method used a subtracting and adding-back techniques widely used in the BEM-based method [13, 14] to regularize the singularities of the kernel functions, so that the source points can be directly located on the physical boundary. The kernel function in the RMM is the double-layer potential because Young et al. [11] consider the desingularization of subtracting and adding-back technique will fail with the single-layer potential. Numerical studies show that the RMM keeps the merit of the MFS and is efficient in the solution of Laplace problem [11], exterior acoustics problem [12] and anti-plane shear problem [15]. However, the double-layer fundamental solution used in the RMM requires the regularization of super-singularities and jeopardizes its solution accuracy as compared with using a single-layer fundamental solution.

Unlike the RMM, Sarler [16] simply uses the single-layer fundamental solution as in the MFS, but unlike the MFS, the method does not require the fictitious boundary and avoids the singularity by using an integral evaluation of its diagonal elements of interpolation matrix. This approach is called the modified method of fundamental solution (MMFS) and has been tested to potential flow problems with modest success. It is noted that the integral calculation makes MMFS more complex and less efficient than the MFS, the BKM and the RMM.

Liu [17] proposed a boundary distributed source (BDS) method, in which the singular fundamental solution is integrated over small areas covering the source points so that the fictitious boundary is circumvented and the coefficients in the system of equations can analytically be evaluated. However, the analytical expression of the diagonal coefficients for equations with the Neumann boundary condition has to be indirectly determined. And thus this method is still immature.

In this study, we focus on a recent boundary-type meshless method, called singular boundary method (SBM), proposed by Chen and his collaborators [18–21]. The SBM overcomes the artificial boundary in the traditional MFS by allowing the source point to coincide with the collocation points on the physical boundary. Its key idea is to introduce the concept of the origin intensity factor to isolate the singularity of the fundamental solution. And an inverse interpolation technique was proposed to